Generic Management

Learner Guide

Module 6 – Mathematical Concepts

FETC Generic Management – Level 4 - SAQA ID: 57712 LP 74630

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The Materials Developer
Ulita Kloke
trainingkzn@gmail.com

Designed and Developed for:

T Wagenaar

Contact: Tracey Wagenaar
1st floor, Virginia Airport Office Block
Virginia, Durban
Tel: 082 093 2416
Email: tracey@bhungane.co.za

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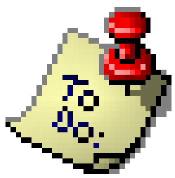
Key to Icons used in this book

We have used symbols (also called icons) throughout the text in this module which can help you quickly identify what you need to do. Here is an example of what each one will require you to do:

Activity



This icon indicates that you are required to complete certain activities that will assist you with your studies.

NB / Note well

This icon indicates information of particular importance.

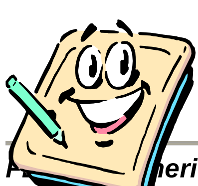
Definition

This icon indicates a clarification of a word/concept or the nature of something.

Portfolio

This icon indicates an activity that can form part of evidence for assessment.

Self Evaluation



If you answer the self-evaluation questions indicated by this icon, you will be able to assess the degree of success you have achieved in mastering the study material.

This module comprises of the following unit standards:

SAQA ID	Unit Standard Title	Level	Credits
9015	Apply knowledge of statistics & probability to critically interrogate & effectively communicate findings on life related problems	4	6
9016	Represent, analyse & calculate shape and motion in 2 and 3 dimensional space in different contexts	4	4
7468	Use mathematics to investigate and monitor the financial aspects of personal, business, national and international issues	4	6

When you have completed this manual you will be able to:

- Critique and use techniques for collecting, organising and representing data.
- Use theoretical and experimental probability to develop models, make predictions and study problems.
- Critically interrogate and use probability and statistical models in problem solving and decision making in real-world situations
- Measure, estimate, and calculate physical quantities in practical situations relevant to the adult with increasing responsibilities in life or the workplace
- Explore analyse and critique, describe and represent, interpret and justify geometrical relationships and conjectures to solve problems in two and three dimensional geometrical situations
- Use mathematics to plan and control financial instruments including insurance and assurance, unit trusts, stock exchange dealings, options, futures and bonds
- Use simple and compound interest to make sense of and define a variety of situations including mortgage loans, hire purchase, present values, annuities and sinking funds
- Investigate various aspects of costs and revenue including marginal costs, marginal revenue and optimisation of profit
- Use mathematics to debate aspects of the national and global economy, including tax, productivity and the equitable distribution of resources.

Unit Standard: 9015
Apply knowledge of statistics & probability to critically interrogate & effectively communicate findings on life related problems

When you have completed this section, you will be able to understand and explain the following:

- ✡ Critique and use techniques for collecting, organising and representing data.
- ✡ Use theoretical and experimental probability to develop models, make predictions and study problems.
- ✡ Critically interrogate and use probability and statistical models in problem solving and decision making in real-world situations.

Specific Outcome 1 & 3:
Critique and use techniques for collecting, organizing and representing data
Critically interrogate and use probability and statistical models in problem solving and decision making in real world situations.

Introduction

Basic Statistics - Terminology

Central Tendency

Mean: The mean is the sum of the scores divided by the number of scores.
Mode: The mode is the score with the highest frequency in a distribution (i.e. the one that occurs most often).
Median: The median is the score that falls at the exact centre of a distribution if the scores are arranged in numerical order.

Population

Defined as the total group of persons or universal collection of items (elements) to which the statistical study/ analysis relates – also called the target population.

Sample

Any subset of the population.

Probability Sample

If each element in a population has a known probability or chance of being included in the sample, the sample is called a probability sample.

Simple Random Sample
The instance where each element of the population that has not yet been included in the sample stands an equal chance of being selected in the next draw.

To apply basic statistical concepts effectively it is necessary to practise them often.

Data types
It is important to understand the nature of data for 2 reasons:
It enables a user to assess data quality and
It enables a user to select the appropriate statistical method to use in order to analyse the data.

Data quality
Three factors influence data quality:

- The type of data collected,
- The source of data collected and
- The methods of data collected

If data quality is poor, irrespective of the sophistication of the statistical method used, confidence in the resulting findings will be low.

Selection of appropriate statistical method
The type of data that is gathered determines the type of analysis, which can be performed on that data. An incorrect application of a statistical method to a particular data type can cause the findings to be invalid.

Data type
The data type is determined by the nature of the random variable, which the data represents.

Random variables are essentially of two kinds:

(1) Qualitative Random Variables
These are variables which yield (give) categorical responses. The data generated by qualitative random variables are classified into one of a number of categories. The numbers, which represent the categories, are arbitrary (i.e. codes).

Example 1

Random Variable	Response Categories	Data codes
Management level	Supervisor	1
	General Manager	2
	Marketing Manager	3
Music Preference (Do you like rock?)	Yes/ No	3 5

(2) Quantitative Random Variables

These are variables that yield numeric responses. The data generated for quantitative random variables can be meaningfully manipulated using conventional arithmetic operations.

Example 2

Random Variable	Response Range	Data
Size of Class	1, 2, 3, 4, 5, 6 ……	e.g. 15 pupils
Distance to Work	0-25km	e.g. 9.4 km

Each random variable category is associated with a different type of data.

There are **two** classifications of **data types**:

Data type 1

Nominal-scaled data
Ordinal-scaled data
Interval-scaled data
Ratio-scaled data

(i) Nominal-scaled data

This data is associated with qualitative random variables. Where data of a qualitative random variable is assigned to one of a number of categories of equal importance, then such data is referred to as nominal-scaled data. There is no ordering between the groups of the random variable.

Example 3

Qualitative Random Response	Variable Categories	Data Codes
Gender	Male / Female	1, 2.
Marital Status	Married / Single Divorced / Widowed	1, 2, 3, 4.
Home Language	English / Zulu French / Italian	1, 2, 3, 4.

Nominal-scaled data is the weakest form of data, since only a limited range of statistical analysis can be performed on such data.

(ii) Ordinal-scaled data

Ordinal-scaled data is mainly associated with qualitative random variables. Like nominal-scaled data, ordinal-scaled data is also assigned to only one of a number of coded categories, but ranking is implied between the categories in terms of being better, bigger, longer, older etc.
Even though there is an implied difference between the categories, this difference cannot be measured exactly. That is, the distance between categories cannot be quantified nor assumed to be equal.

Example 4

Qualitative Random Variable	**Response Categories**	**Date Codes**
T-shirt size	small / medium / large	1, 2, 3
Management levels	lower / middle / senior	1, 2, 3
Magazine type	"Rank the top 3 magazines you most often read from the following list"	1, 2, 3

(iii) Interval-scaled data

This data is associated with quantitative random variables. Differences can be measured between values of a quantitative random variable. Thus interval-scaled data possesses both order and distance properties.
Because interval-scaled data does not possess an absolute origin the ratio of values cannot be meaningfully compared.

Example 5

Indicate your response to the statement "Driving is a pleasant experience for me". (Tick a value from the Likert Rating Scale)

Strongly Disagree	**Disagree**	**Unsure**	**Agree**	**Strongly Agree**
1	2	3	4	5

In social research studies, the Likert Rating Scale – which asks respondents to indicate a preference or a perception on a scale, which can range from 1 to 5 (or even 1 to 7) – is often assumed to possess interval-scaled properties. Preferences or perceptions are assumed to be measured on a continuum from one extreme position to the opposite extreme position. On a 1 to 5 Likert rating scale, a 4 is considered to reflect a stronger perception (or preference) than a 2. The differences between the ratings are assumed to reflect equal differences between perceptions or expressed preferences. It is possible to conclude however, that the difference in perception (or preference) between 3 and 4 is the same as between 1 and 2. This is the property of distance. In general, the wider the rating scale, the more valid the interval-scaled assumption becomes.
(This explanation has been taken from: Applied Business Statistics Methods and Applications. Trevor Wegner)

Interval-scaled data is most often generated in marketing studies through rating responses on a continuum scale.
A wide range of statistical techniques can be applied to interval-scaled data as it possesses numeric (measurement) properties.

(iv) Ration-scaled data

This data is associated with quantitative random variables. 'If the full range of arithmetic operations can be meaningfully performed on the observations of a random variable, the data associated with that random variable is termed ratio-scaled'. It is numeric data with a zero origin. The zero origin indicated the absence of the attribute being measured.

Example 6

Quantitative Random Variable	Examples of Data Values
Time	26 min
Age	21 years
Income	R5 987
Distance	143 km
Mass	450 g
Prices	R4,40

It should be noted that if ratio-scaled data is grouped into categories, the data type becomes ordinal-scaled. This then reduces the scope for statistical analysis on the random variable.

Example 7

Random Variable	Response Categories	Data Code Used
Age	0-10	1
	11-20	2
	21-30	3
	31-40	4
	41-50	5

By capturing age data in categories instead of in terms of actual ages, the data becomes ordinal-scaled. However, the random variable remains quantitative in nature.

When data capturing instruments are set up, it is important to ensure that the most useful form of data is captured. This is not always possible for reasons of convenience, cost and sensitivity of information. This applies particularly to random variables such as age, personal income, company turnover and consumer behaviour questions of a personal nature.

Data Type 2

A second classification of data types is between:
- Discrete data, and
- Continuous data

Discrete data
Discrete random variables are random variables which take on only specific values, usually only integer (whole number) values. In such instances, certain values are valid, while others are invalid.

Examples of random variables generating discrete data
- The number of students in a class
- The number of employees in an organization
- The number of shampoos on a shelf in a store
- The number of cars bought in a year

Continuous data
Continuous data is a random variable which can take on any value in an interval
- The speed of a car
- The age of an antique
- The length of a ruler
- The mass of a pocket of cement
- The time taken to walk 10 km's
- The strength (tensile) of material

Data sources
Data for statistical analysis are available from various sources.
There are two classifications of data sources:
- Internal / external sources, and
- Primary / secondary sources

1. Internal versus external sources

1.1 Internal data sources
This refers to the availability of data from within an organization.

1.2 External data sources
Data that is available from outside an organization.
These sources may be private institutions, trade / employer / employee associations, profit motivated organizations, or governmental bodies.

2. Primary versus secondary

2.1 Primary Data Sources

If data is captured for the first time and with a specific purpose in mind.

Advantages:

- It is directly relevant to the problem at hand and
- It generally offers greater control over data accuracy

Disadvantages:

- Primary data is time consuming to collect
- Primary data is generally more expensive to collect

2.2 Secondary Data Sources

Is data, which already exists either within or outside an organization.

Advantages:

- The data already exists
- Access time is relatively short
- Secondary data is usually less expensive to obtain

Disadvantages:

- Data may not be problem specific
- Data may be dated and therefore inappropriate
- It may be difficult to assess data accuracy
- Data may not be subject to further manipulation
- Combining various sources could lead to errors of collation and introduce bias

Data collection methods

There are three techniques or methods used to collecting data.

1. Direct observation
2. Interviewing
3. Experimentation

1. Observation Methods

a. Direct observation

Primary data can be collected by directly observing the respondent or item in action.

Examples of direct observation approaches are:

- Vehicle traffic surveys
- Pedestrian flow surveys
- Observing purchase behaviour of brands in a store

- Observing work practices in a production process
- Quality control inspection

Advantages:

- The people being observed are not aware and therefore behave in a natural way. This ultimately reduces the likelihood of gathering biased data.

Disadvantages:

- It is a passive form of data collecting therefore investigating behaviour further is not possible.

b. Desk research

Another form of data gathering through observation is consulting and extracting secondary data from a number of source documents. A number of organizations and individuals continually use secondary data for decision making or opinion forming. These organizations and individuals include economists, financial planners, politicians, senior executives, research groups, and scenario planners.

Advantages:

- The data are already in existence
- Access time is relatively short
- The data are generally less expensive to acquire

Disadvantages:

- Data may not be problem specific
- Data may be dated and hence inappropriate
- It may be difficult to assess data accuracy
- Data may not be subject to further manipulation
- Combining various sources could lead to errors of collation and introduce bias.

2. Interview methods

There are three approaches to gathering interview data.

a. Personal interviews

In person interviews, questionnaires are complete through face-to-face contact with the respondent.

Advantages:

- A higher response rate is achieved
- Questioning allows probing for reasons
- Data collection is immediate

- Greater data accuracy is generally ensured
- Useful when response data of a technical nature is required
- Non-verbal responses can be asked
- Responses are spontaneous
- The use of aided-recall questions is possible

Disadvantages:

- Personal interviews are very time consuming
- It requires trained interviewers, and is therefore more expensive
- Fewer interviews are conducted because of cost and time limits

b. Postal surveys

Postal surveys are used when the target population is large and/or geographically dispersed.

Advantages:

- A larger sample of respondents can be reached
- Postal interviews are more cost effective
- Interviewer bias is eliminated
- Respondents have more time to consider their responses
- The person filling in the questionnaire remains anonymous
- Respondents are more willing to answer personal questions

Disadvantages:

- Response is very low
- The respondent cannot obtain clarity on questions
- Questionnaires must be shorter and simpler
- Possibilities of probing or investigating further are limited
- Data collection takes a long time
- There is no control over who answers the questionnaire

c. Telephone interviews

An interview is conducted telephonically with a respondent.

Advantages:

- It allows quicker contact with geographically dispersed respondents
- Call-backs can be made if the respondent is not initially available
- Cost is low
- People are more willing to talk on the telephone
- Interviewer probing is possible
- Clarity on questions can be provided by the interviewer
- Use of aided-recall questions is possible

Disadvantages:

- Respondent anonymity is lost
- Non-verbal responses cannot be observed
- Trained interviewers are required, thus increasing costs
- The respondent may terminate the interview simply by putting the telephone down
- Sampling errors are compounded if significant numbers of the target population do not have telephones

Questionnaire design
The questionnaire is the data collection instrument used to gather data in all interview situations. The way in which the questionnaire is designed is critical to ensure that the correct research questions are addressed and that accurate and appropriate data for statistical analysis is collected.

A questionnaire consist of three sections

a. The administrative section
This is used to record the identity of the respondent and the interviewer by name, date and address and where the interview was conducted. It should also contain an interview number.

b. The demographic section
This section describes the respondent by a number of demographic characteristics, (which include, age, gender, residential location marital status, language, qualifications etc.) for individual respondents; or company size, economic sector, managerial level, functional area, etc. for company respondents. The selection of demographic characteristics should be dictated by the nature and purpose of the research

c. The information sought section
Makes up the major portion of the questionnaire and consists of all the questions which will extract data from respondents to address the research objective.

3. Experimentation
Primary data can be generated through the manipulation of variables under controlled conditions. Data on the primary variable under study can be monitored and recorded while conscious efforts are made by the researcher to control the effects of a number of influencing factors.

Advantages:

- Quality, “noise-free” data is collected on the research problem if the experiment is correctly designed and executed.

- The results of such studies are generally more objective and valid

Disadvantages:
- It is costly and time consuming
- It may be impossible to control certain extraneous factors which can confound the results.

Sampling
It is difficult to gather all the data on a random variable under study for analysis purposes. Therefore a subset of all observations, called a sample, is collected.

Sampling Methods
Sampling is the process of selecting a representative subset of observations from a population to determine the characteristics of the random variable under study.

There are two basic methods of sampling:
- Non-probability sampling methods
- Probability sampling methods

Non-probability Sampling
Any sampling method in which the observations are not selected randomly is called non-probability sampling.
There are 3 types of non-probability sampling techniques:

1. Convenience Sampling

This represents a sample drawn to suit the convenience of the researcher. For example, it is more convenient to gather data on shopping habits from only one shopping centre than it is from 7 different shopping centres.

2. Judgement Sampling

This sampling is used by researchers to select the best sampling units to include in the sample. For example, only beauticians instead of all women users of cosmetics may be selected to provide information on skin care practices.

3. Quota Sampling

The population is divided into segments and a quota of observations is collected from each segment.

The disadvantage of non-probability sampling methods is the unrepresentative nature of the sample with respect to the population from which it is drawn. This type of sampling is likely to produce biased results because entire sections of the population are omitted from the selection process.

Another disadvantage of non-probability sampling is that the sampling error cannot be quantified.
Consequently results based on non-probability sampling would be invalid.
However, non-probability samples can be useful in exploratory research to obtain initial impressions of the characteristics of a random variable under study.

Probability Sampling Methods
Probability sampling includes all selection methods where the observations to be included in a sample have been selected on a purely random basis from the population.

There are 4 methods of randomly selecting observations:

1. Simple Random Sampling
Each observation in the entire population has an equal chance of being selected. This method is used when it is assumed that the population is relatively homogeneous with respect to the random variable under study.

2. Systematic Random Sampling
Sampling begins by randomly selecting the first observation. Thereafter, subsequent observations are selected at a uniform interval relative to the first observation.

Example: The first name from a mailing list of a company is randomly selected. Thereafter, subsequent company names are selected systematically, say every 4^{th}, 8^{th} or 16^{th} name.

3. Stratified Random Sampling
If the population is seen as being heterogeneous with respect to the random variable under study, the population can be divided into segments where the sampling units in each social level are relatively homogeneous. Thereafter, random samples are selected from each segment.

4. Cluster Random Sampling
The population is divided into clusters, where each cluster is similar in profile to every other cluster. Once the clusters have been formed clusters are then randomly selected for sampling. The sampling units within these randomly selected clusters may then be randomly selected to provide a representative sample from the population.
Cluster sampling tends to be used when the population is large and spread out over a geographical area.

Graphic Representation of Statistical Results

Graphical techniques are commonly used to convey statistical results. Graphs can display findings concisely, clearly and in an easy-to-understand format.
In practice, an analyst should always consider using graphical techniques ahead of written methods to promote rapid understanding of statistical information by managers.

Data from qualitative random variables (i.e. nominal and ordinal scaled data) are summarised into table format by counting the number of observations within each category of the random variable.

These category totals can then be compared and contrasted graphically using one of the following charting techniques:

- Pie Chart
- Simple Bar Chart
- Multiple Bar Chart

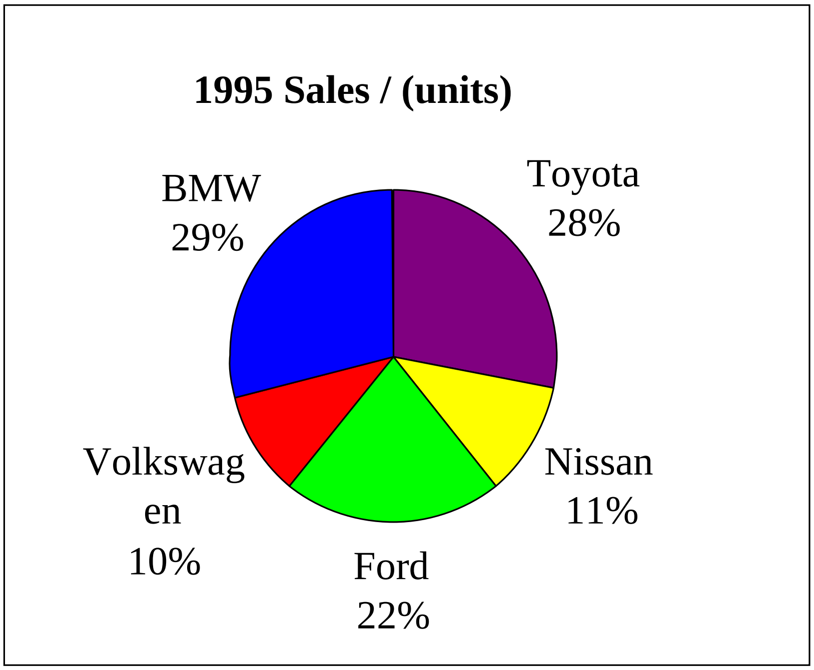
Pie Chart
A pie chart is a circle divided into segments. The size of each segment is proportional to the importance or frequency of the data category of a random variable relative to the whole and is usually expressed in percentage terms.
The size of each segment reflects the relative importance of each category of a random variable.
Pie charts are best suited to display categorical data. Continuous data can also be displayed in pie chart form. But it must first be grouped into intervals.

Example 8

With the following information provide a pie chart showing the percentage market share of the South African passenger car market held by each of South Africa's car manufacturers.

Manufacturer (Random Variable)	**1995 Sales / (units) (Absolute frequency)**	**Percentage (Relative Frequency)**
Toyota	51 653	27.9%
Nissan	20 793	11.3%
Ford	39 757	21.5%
Volkswagen	18 631	10.1%
BMW	53 897	29.2%
Total	**184 731**	**100%**

Solution
A pie chart showing the percent market share of the South African passenger car market interpretation:



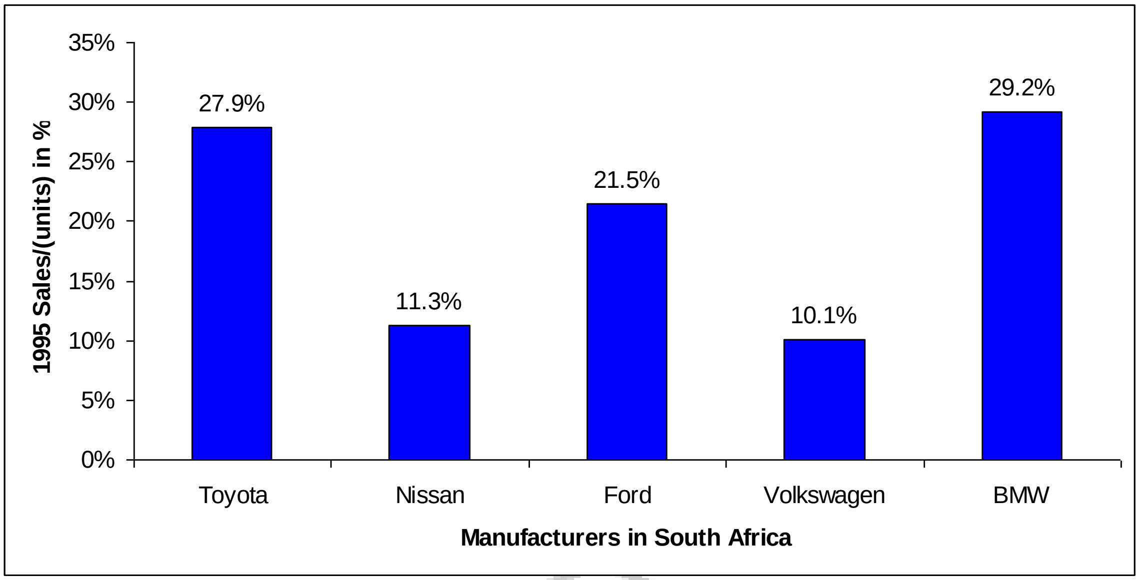
The pie chart above shows that BMW has the largest percentage of market share in the South African passenger car market. And that Volkswagen has the smallest percent of the market share in South Africa.

Note: All diagrams must be adequately labelled with titles and legends to avoid misinterpretation, and the source of the data must be identified.

Bar Chart
This simple bar chart displays the same information as a pie chart, however the display is in a bar form instead of segments of a circle. The height of each component bar is proportional to the importance or frequency of the random variable component in relation to the whole.

Example 9

Construct a bar chart using example 8 data to show the percent market share of the South African passenger car market held by each of South Africa's car manufacturers.



Note:

- The categories of the random variable are displayed on the horizontal axis (x-axis).
- The vertical axis (y-axis) can be expressed either in:
    - ➢ percentage terms (i.e. showing the percentage of respondents per residential area), or in
    - ➢ absolute terms (i.e. the number of observations is given as the frequency)
- The height of each bar is important. The sum of the heights of all the bars must:
    - ➢ equal 100%, if expressed as percentages, or
    - ➢ equal the total number of observations, if expressed in absolute terms.
- The width of the bars must be of equal thickness to avoid distortion of category importance's, but neither the order of categories on the x-axis nor the actual choice of bar thickness is important. A limitation of the simple bar chart is that it can display information on only one random variable at a time.

NOTES

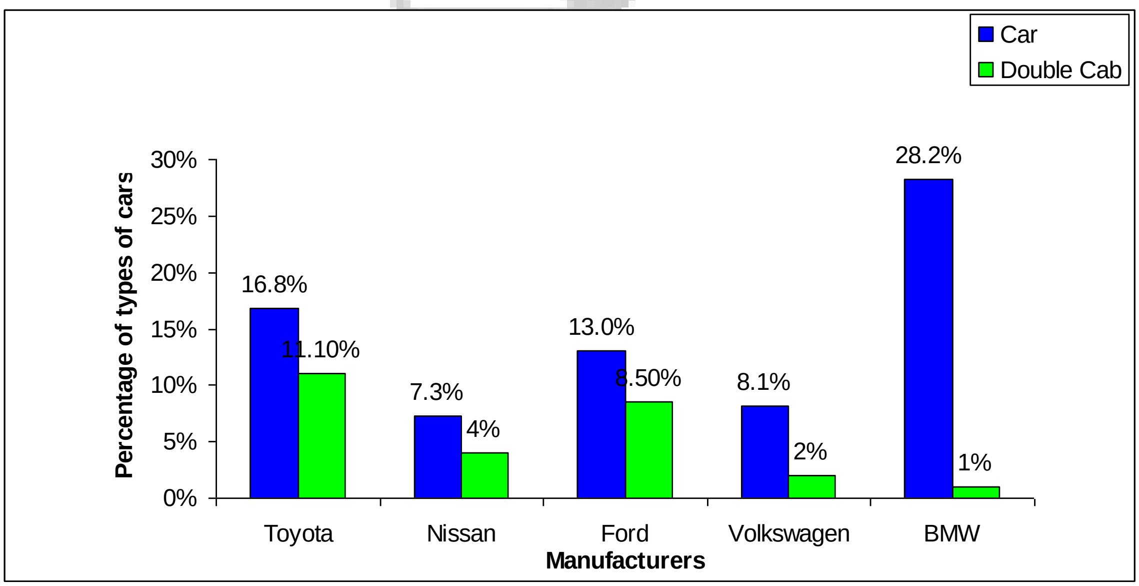
Multiple Bar Charts

When trying to emphasis the difference between the categories of one random variable for each category of a second random variable, then a multiple bar chart is an appropriate graphical display.

Example 10
Draw a multiple bar chart with the information provided below.

Manufacturer (Random Variable)	Car Type		Percentage (Relative Frequency)
	Car	**Double Cab**	
Toyota	16.8%	11.1%	27.9%
Nissan	7.3%	4%	11.3%
Ford	13%	8.5%	21.5%
Volkswagen	8.1%	2%	10.1%
BMW	28.2%	1%	29.2%
Total	**73.4%**	**26.6%**	**100%**

A multiple bar chart showing the percentage market share of the South African passenger car type held by each of South Africa's car manufacturers



Interpretation: This chart shows that BMW may have the most market share in South Africa with regards to total market share however, BMW has little or no market share in the double cab industry; neither does Volkswagen. Toyota has a reasonable market share for both car and double cab markets.

Measures of Central Location
Numbered measures (called statistics) convey more precise information about the behaviour pattern of a random variable.

The behaviour pattern of any random variable can be described by:

- A measure of central location and
- A measure of spread of observations about this central value

A central location statistic represents a typical value or middle data point of a set of observations and are useful for comparing data sets.

There are three (3) main measures of central location:

- Arithmetic mean (or arithmetic average),
- Mode, and
- Median (also called the second quartile or the 50^{th} percentile).

Each measure will be computed for **ungrouped data** (raw data) only.

a. Arithmetic Mean (Arithmetic Average)
(ungrouped data method)

The arithmetic mean is defined as:

$$\bar{x} = \frac{\text{sum of all observations}}{\text{number of observations}} = \frac{\text{sum of all observations}}{n}$$

Example 11
A restaurant owner randomly selected and recorded the value of meals enjoyed by 20 diners on a given day. The values of meals were (in rand)

28	23	29	28	29	27	26	28	29	30
29	27	28	26	30	29	28	27	26	28

Find the arithmetic mean of the value of meals enjoyed by 20 diners.

Solution

$$\bar{x} = \frac{28+23+29+28+...+28+27+26+28}{20}$$

$$\bar{x} = \frac{555}{20}$$

$$\bar{x} = \text{R52.15}$$

Interpretation:
On average the value of a meal enjoyed by 20 diners is R52.15

b. Mode
(ungrouped data method)

The mode is defined as the most frequently occurring value in a dataset.
The mode is seldom computed for ungrouped data. Since the "more frequently occurring" value is more easily identified from frequency counts, it is usually only found once the data has been grouped into a frequency distribution.
However, if the number of observations is not too large, the mode can be found by arranging the data in ascending order, and by inspection, identify that value that occurs most frequently.

Example 12

From Example 11 determine the mode.

Solution

23	26	26	26	27	27	27	28	28	28
28	28	28	29	29	29	29	29	30	30

The mode is equal to 28

c. Median
(ungrouped data method)

The median is that value of a random variable which divides an ordered dataset into two equal parts.
Half the observations will fall below this median value and the other half above it.

Step 1: Arrange the observations in ascending order

Step 2:

- If **n** is **odd**:

Identify the median position by the formula $\frac{n+1}{2}$

The value in this position is the median

Example 13
For the unordered dataset of n = 9 observations:

25	23	21	27	25	29	21	26	21

The ordered dataset becomes:

21	21	21	23	25	25	26	27	29

The median position $= \frac{9 + 1}{2} = 5$

The value which occupies the 5^{th} position is the median, i.e. Median = 25.

- If **n** is **even**:

Identify the value in the $\frac{n}{2}$ position.
Average this value and the adjacent value on its right to find the median value

Example 14
For the unordered dataset of n = 10 observations:

28	23	27	23	24	21	29	22	26	25

The ordered dataset becomes:

21	22	23	23	24	25	26	27	28	29

For the $\frac{n}{2}$ position: $\frac{n}{2} = \frac{10}{2} = 5$ (i.e. 5^{th} position)

Now average the values in the 5^{th} and 6^{th} position, i.e.

Median $= \frac{24 + 25}{2} = 24,5.$

Measures of Spread (Dispersion) Location
Above it was mentioned that there are two descriptive statistical measures which are always used to describe the behaviour (or characteristics) of a random variable. They are:

- A measure of central location (refer above); and
- A measure of spread or dispersion.

Spread (or dispersion) refers to the extent by which the observations of a random variable are scattered about the central value.

Measures of dispersion provide useful information with which the reliability of the central value may be judged. Widely dispersed observations indicate low reliability and less representativeness of the central value. Conversely, a high concentration of observations about the central value increases confidence in the reliability and representativeness of the central value.

The measures that are used to describe dispersion are:

- Range
- Inter-quartile Range
- Quartile Deviation
- Variance
- Standard Deviation

The method of computation, appropriate data types, uses and interpretation of each are now described.

a. Range

The range is the difference between the highest and lowest observed values in a dataset.

Range = Maximum value – Minimum value

= $x_{\max} - x_{\min}$ for ungrouped data; or

= Upper-limit (highest class) – Lower-limit (lowest class) for grouped data

The re-order problem..
(data obtained from Example 12)

23	26	26	26	27	27	27	28	28	28
28	28	28	29	29	29	29	29	30	30

$x_{\max}$ = 30

$x_{\min}$ = 23

Range = 30 – 23 = R7.00

The range is a crude estimate of spread. It is easily calculated, but is distorted by extreme values (outliers). An outlier would be x min or x max. It is therefore a volatile and unstable measure of dispersion as it can vary greatly between samples taken from the same population. It also provides no information on the clustering of observations within the dataset about a central value as it uses only two observations (i.e. x min and x max) in its computation.

b. Inter-quartile Range

Because the range can be distorted by extreme values, a modified range which excludes these outliers is often calculated. This modified range considers the variability shown by only the middle 50 per cent of observations and is called the inter-quartile range.

It is the difference between the upper and lower quartiles.

Inter-quartile Range = Q3 – Q1

This range removes some of the instability inherent in the range if outliers are present, but it excludes 50 per cent of all observations from further analysis. This measure of dispersion, like the range, also provides no information on the clustering of observations within the dataset as it uses only two observation (i.e. Q1 and Q3) in its computation.

c. Quartile Deviation (QD)

A measure of variation based on this modified range is called the quartile deviation. It is found by dividing the inter-quartile range in half.

$$\text{Quartile Deviation (Q.D)} = \frac{Q3 - Q1}{2}$$

Quartile deviation for the re-order problem
(Data from Example 11)

Each quartile position is determined as follows:

$$Q1 = \frac{n}{4} = 5$$

$$Q2 = \frac{n}{2} = 10$$

$$Q3 = \frac{3n}{4} = 15$$

$$\text{Quartile Deviation} = \frac{15 - 5}{2} = 5$$

The quartile deviation is an appropriate measure of spread for the median. It identifies the range below and above the median within which 50 per cent of the observations are likely to fall.

The quartile deviation is useful as a measure of dispersion if the sample of observations contains excessive outliers as it ignores the top 25 per cent and bottom 25 per cent. Quartile deviation does not use all the observations and therefore gives no indication of the spread of values between the upper and lower quartiles.

d. Variance

The most useful and reliable measures of dispersion are those that:
Take every observation into account, and
Are based on an average deviation from a central value

The variance (denoted S^2) is such a measure of dispersion.

By having these properties, the variance statistic has become the most commonly used measure of dispersion. It is a powerful statistic which is used extensively in statistical analysis.

Computation for ungrouped data

(This example has been taken from Trevor Wegner: Applied Business Statistics)
Illustration – consider the ages (in years) of 7 second hand cars:
13 7 10 15 12 18 9 (in years)

Step 1

Find the sample mean. $\bar{x} = \frac{84}{7}$ 12 years

Step 2

Find the squared deviation of each observation from the sample mean.
The sum of the deviations is zero, this will always be true when deviations about the mean are measured and summed. It emphasises the unbiased nature of the arithmetic mean.
Note: the deviations must first be squared to avoid plus and minus deviations cancelling each other. These squared deviations are then summed.

Table
Variance calculation for the Car Ages problem

Car Ages x_i	Sample Mean $\bar{x}$	Deviation $(x_i - \bar{x})$	Squared Deviations

			$(x_i - \bar{x})^2$
13	12	+1	1
7	12	-5	25
10	12	-2	4
15	12	+3	9
12	12	0	0
18	12	+6	36
9	12	-3	9
		$\sum_{i=1}^{n}(x_i - \bar{x}) = 0$	$\sum_{i=1}^{n}(x_i - \bar{x})^2$ =84

Step 3
Determine a measure of average squared deviation/
A measure of average squared deviation can now be found by dividing the total squared deviation by (n - 1). It is called the variance, i.e.:

$$S_x^2 = \frac{84}{(7-1)}$$

$$= 14 \text{ years}^2$$

Note: Division by n would appear logical, but the variance statistic would then be a biased measure of dispersion. It can be shown to be unbiased if division is by (n – 1). For larger samples however this distinction becomes less important.

The formula for a variance can now be expressed as:

$$\text{Variance} = \frac{\text{Sum of squared deviation}}{(\text{Sample size} - 1)}$$

$$= \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{(n-1)}$$

Regression and Correlation Analysis
Regression analysis and Correlation analysis are two statistical methods which attempt to quantify and describe this possible relationship between variables.

“Regression analysis is concerned with quantifying the underlying structural relationship between variables, while

Correlation analysis examines the strength of this identified association between variables."

Simple Linear Regression Analysis
This aims to find a linear relationship between the values of two random variables only.
One of the random variables is termed the **independent variable (x)**. It is the variable for which values are known or easily determined.

The other random variable is termed the **dependent variable (y)**. Values for this variable are not readily known and need to be estimated from values of the independent variable (x).

In simple linear regression only one independent variable is used to estimate values of the dependent variable y.

Example 15

Independent Variable (x) (potential predictors of y)	Dependent variable (y) (variable to be estimated)
Speed	Fuel Consumption
Hours worked	Machine Output
Hours in Training	Productivity
Years of Experience	Number of Sales
Number of Calls	Sales Turnover

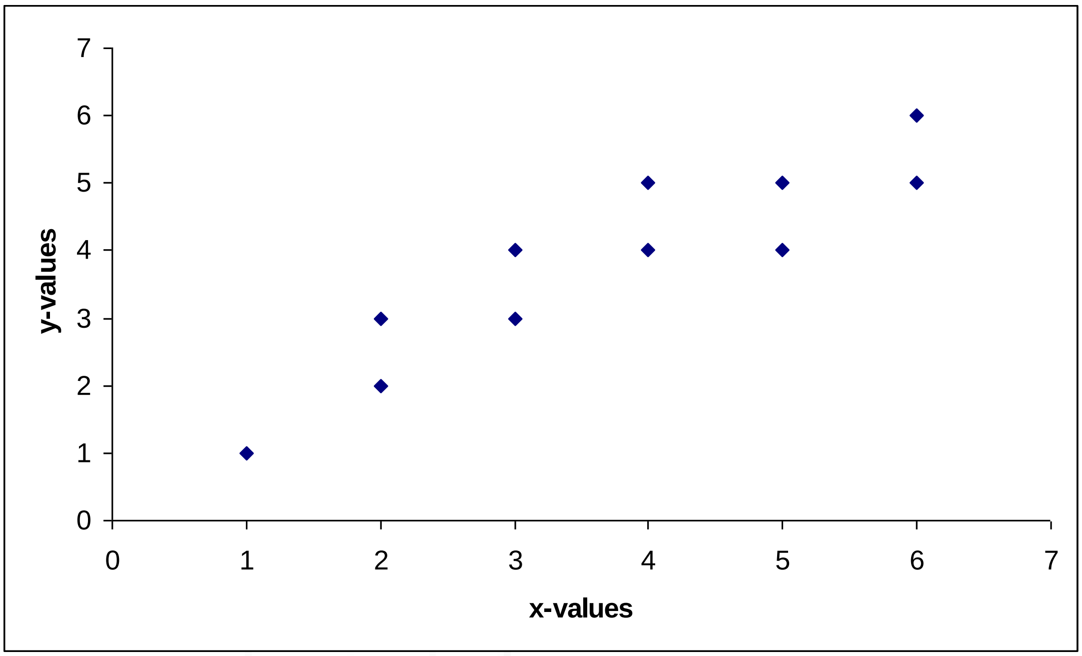
Scatter Plot
A scatter plot is a graphical plot of the values of the independent and dependent variables. The x values are recorded along the horizontal axis and the y values along the vertical axis.

Pairs of x and y observations are plotted in space.
A visual inspection of the likely relationship between the two variables x and y, which is shown in a scatter plot, will provide an insight and understanding into the likely regression and correlation analysis results.

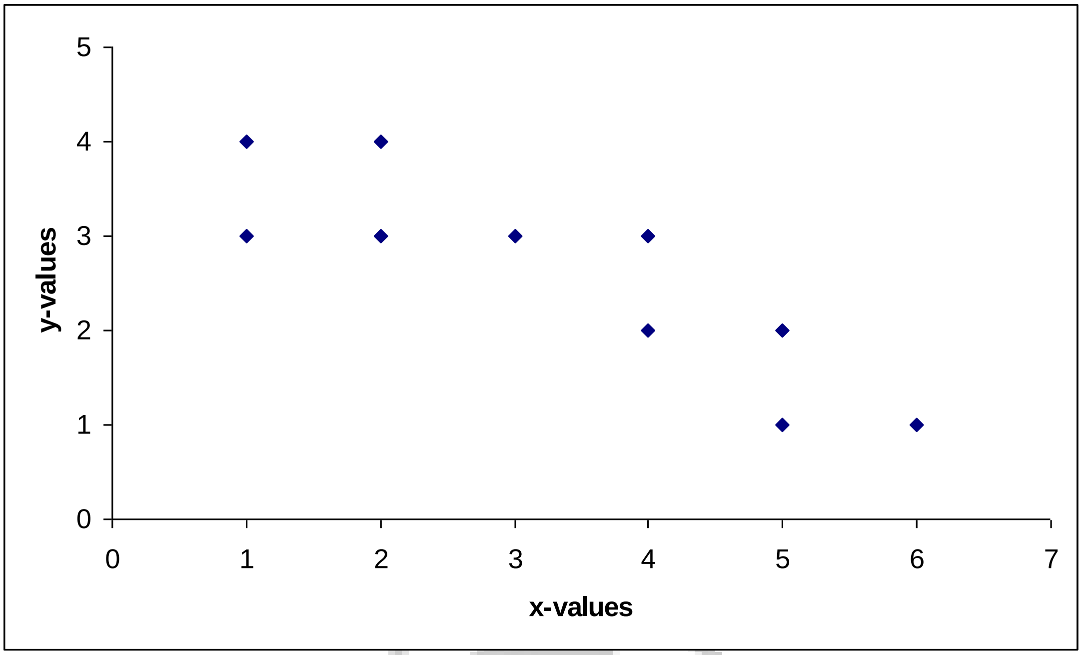
Note: If the data points are widely scattered and the range of the y values is large for any given x value, thin a linear regression function will be of little value as an estimation function for y. The correlation measure too will show almost no association.

ILLUSTRATIONS

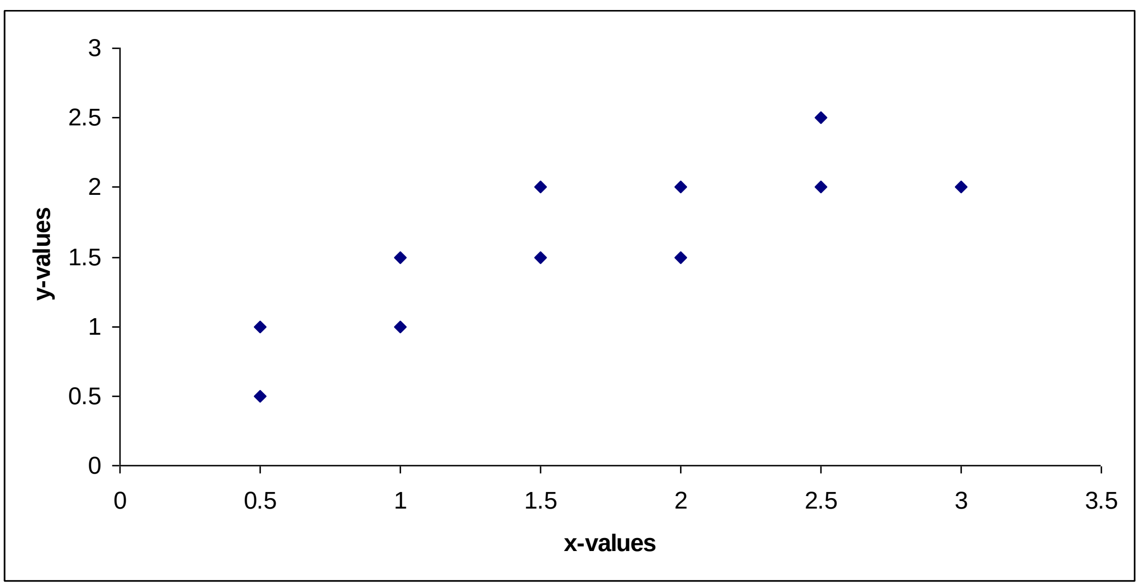
i. Direct linear relationship with small dispersion
(for any given x value the range in y values is small)



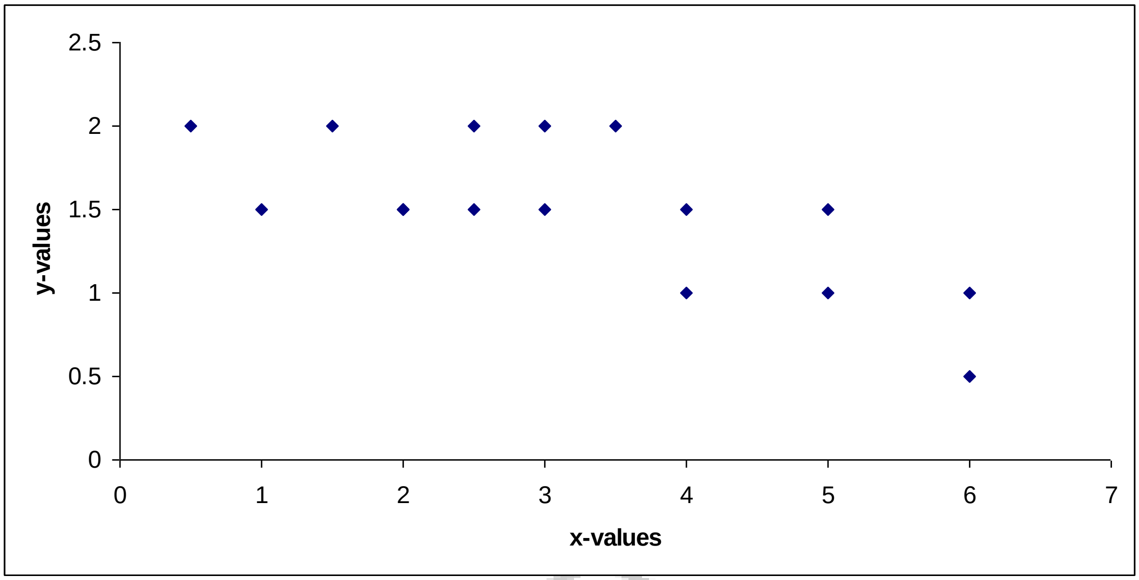
ii. Inverse linear relationship with small dispersion
(for any given x value, the range in y values is small)



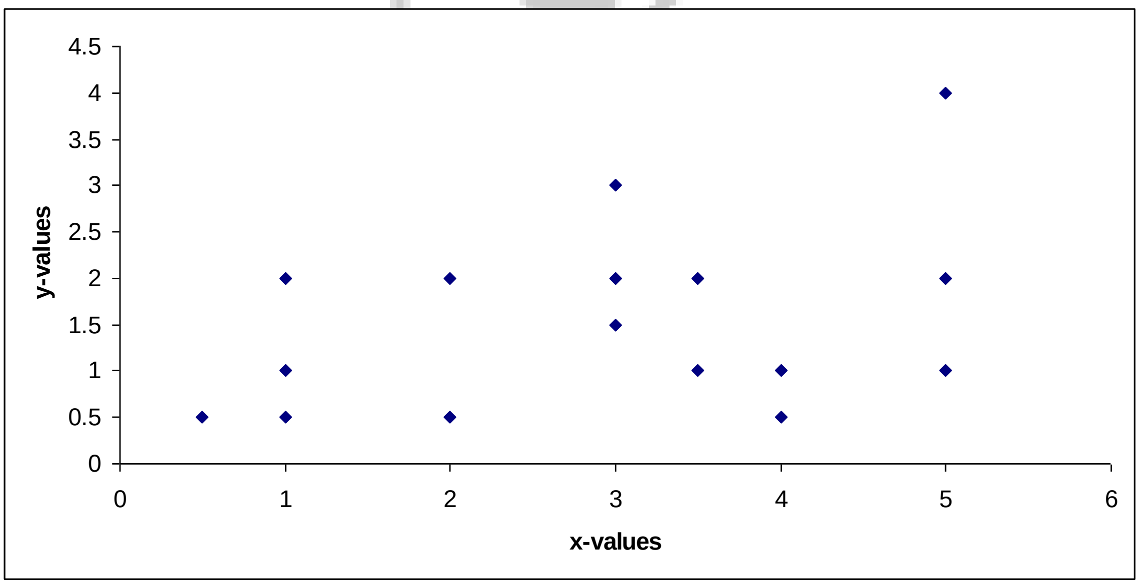
iii. Direct linear relationship with greater dispersion
(for any given x value, the range in y values is larger)



iv. Inverse linear relationships with greater dispersion
(for any given x value, the range in y values is larger)



v. No linear relationship (values of x and y are randomly scattered)
(for any given x value, y can have any value over a wide range)



Calculating the Linear Regression Function
Regression analysis aims to find a linear (i.e. a straight line) function which best fits the actual observations.

A straight line graph is defined as follows:

ŷ = a + bx

Where; x = values of the independent variable
ŷ = estimated values of the dependent variable
a = the y intercept (where the regression line cuts the y axis)
b = the slope of the regression line (for every unit change in x, y changes by b units)

Finding the a and b values
Without providing the derivation, the coefficients a - and b which results from the method of least squares are given as follows:

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$a = \frac{\sum y - b\sum x}{n}$$

The values of a and b found from the above formulae define the best fit linear regression line.

Correlation Analysis
The reliability of the estimate of y depends on the strength of the relationship between the x and y variables. A strong relationship implies a more reliable estimate of y.

Correlation analysis measures the strength of a linear association between x & y.
This statistical measure is called correlation coefficient.
There are two commonly used measures of correlation:

- Pearson’s correlation coefficient, and
- Spearman’s rank correlation coefficient

We however will only be looking at Pearson’s Correlation.

Pearson’s correlation coefficient computes the correlation between two ratio scaled random variables.
Spearman’s rank correlation finds the correlation between two ordinal scaled (i.e. ranked data) random variables.

Pearson’s correlation coefficient (r) is defined by:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - \left(\sum x\right)^2] \times [n\sum y^2 - \left(\sum y\right)^2]}}$$

Where;

r= the correlation coefficient
x= the values of the independent variable
y= the values of the dependent variable
n= the number of paired data points

The correlation coefficient is a proportion which takes on values only between -1 and +1.

Example 16

Bulk advertising is used to influence buyer response. For example, suppose a product sells for R0.50. If it is advertised at 2 for R1.00 or at 4 for R2.00 some people think they are getting a bargain. To test this theory, a store manager advertised an item for equal periods of time at five bulk rates and observed the data listed below.

Advertised number In bulk sale	Volume sold
1	38
2	47
3	45
4	74
5	63

a. Compute the least squares regression equation.
b. Calculate the correlation coefficient.

Solution

Find the regression coefficients a and b.

Advertising numbers in bulk sale	Volume sold	x^2	xy	y^2
1	38	1	38	1444
2	47	4	94	2209
3	45	9	135	2025
4	74	16	296	5476
5	63	25	315	3969
$\sum x = 15$	$\sum y = 267$	$\sum x^2 = 55$	$\sum xy = 878$	$\sum y^2 = 15123$

a. Compute the least squares regression equation y=a + bx

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b = \frac{5(878) - (15)(267)}{5(55) - (15)^2}$$

$$b = \frac{4\,390 - 4005}{275 - 225}$$

b = $\frac{385}{50}$

b = 7,7

a = $\frac{\sum y - b\sum x}{n}$

a = $\frac{267 - (7,7)(15)}{5}$

a = $\frac{267 - 115,5}{5}$

a = $\frac{151,5}{5}$

a = 30,3

Therefore:

ŷ =a + bx

ŷ =30,3 + 7,7x

b. Calculate the correlation

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2] \times [n\sum y^2 - (\sum y)^2]}}$$

$$r = \frac{5(878) - 15(267)}{\sqrt{[5(55) - (15)^2] \times [5(15123) - (267)^2]}}$$

$$r = \frac{4390 - 4005}{\sqrt{(275 - 225) \times [(75615) - (71289)]}}$$

r = $\dfrac{385}{\sqrt{50 \times 4326}}$

r = $\dfrac{385}{\sqrt{216300}}$

r = $\dfrac{385}{465.0806382}$

r = 0.8278

Group Exercise 1

Learners will break into groups and perform the following tasks:

1. Learners will identify, discuss and document different data collection methods, and which method best suits which task. The discussion must include examples based on direct observation, interviews, and experimentation

2. Identify at least 4 different situations or issues that can be dealt with using statistical methods. Explain and document each situation and reasons for it's suitability.

3. Discuss and document a number of different methods of collecting, recording, and organising data. The discussion should include direct observation, interview techniques, and experimentation.

4. Demonstrate how to select data sources and databases in such a way as to ensure that data is valid and correctly represented.

Group Exercise 2

Sample data will be handed to the group by the facilitator. The group will identify whether the data provided has any form of bias or contamination, and if so, identify the contamination and the effect that it has on the data. The explanation should be documented in writing

Group Exercise 3

Groups will identify a subject on which they would like to research and gather data. They will draw up a suitable questionnaire to gather the data. Data is researched, gathered, sorted, and then represented statistically in a numerical summary

Group Exercise 4
Areas of continents of the world

Continent	Area in millions of squared kilometres (km^2)
Africa	40.2
Australia	20.1
North America	30.6
Europe	10.9
Asia	32.3
South America	25.6
Oceania	12.5

(i) Draw a bar chart of the above information
(ii) Construct a pie chart to represent the total area

Learners must support the situation or issue through the data collected, and their representation of it.

Group Exercise 5
Employee bonuses earned by workers at a steel factory in a recent month (in Rands) were:

50	37	72	81	43	68	58	29	45	75	50	61
28	39	29	50	46	53	73	31	47	50	35	62

(i) Find the arithmetic mean, mode and median of the ungrouped data.
(ii) Find the range, inter-quartile range and quartile deviation

Group Exercise 6
Your facilitator will provide learners with statistics on the population living with HIV/AIDS. Learners must collate and extract meaningful data from the statistics provided, and then show summations in graphical or other formats .

Using the data generated, learners must:

- Show how assumptions made in the collection and generation of data and statistics are defined or critiqued appropriately
- Demonstrate how tables, diagrams, charts, and graphs developed from the data are used or critiqued in the analysis or representation of data
- Show how predictions, conclusions and judgements are made on the basis of valid arguments, statistics and probability models

Specific Outcome 2:
Use theoretical and experimental probability to develop models

What is a Probability?

In general a probability is the chance that something will happen. Probability is a measure of the likelihood that an event in the future will happen; it can only assume a value between 0 and 1, inclusive.

When studying probability there are three key words: **experiment**, **outcome** and **event.**

Experiment: The observation of some activity or the act of taking some measure. In reference to probability, an experiment has two or more possible results and it is uncertain which will occur.

Outcome: A particular result of an experiment.

For example, the tossing of a coin is an experiment. You may observe the toss of the coin, but you are unsure whether it will come up “heads” or “tails”. When one or more of the experiment’s outcomes are observed, we call this an event.

Event: A collection of one or more outcomes of an experiment.

The example below is an example to clarify the definitions of the terms experiment, outcome and event.

Experiment:	Roll a die
Possible Outcomes:	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6

Possible Event: Observe an event number
Observe a number greater than 4
Observe a number 3 or less

In the die rolling experiment there are six possible outcomes, but there are many possible events.

How is Probability Expressed?
A probability is expressed as a decimal, such as .50, .23, .67or .99. However, it may also be expressed or given as a fraction. The closer a probability is to 0, the more improbable it is that something will happen. The closer the probability is to 1, the more, sure we are it will happen.

Types of Probabilities
Probabilities are broadly of two types:

- Subjective
- Objective

Subjective Probability
Subjective probability is where the probability of an event is based on an educated guess or expert opinion. Subjective probabilities cannot be statistically verified. They are not used extensively in statistical analysis and will not be considered further.

Objective Probability
Objective probability is when the probability of an event can be verified, usually through repeated experimentation or empirical observations. It is this type of probability that is used extensively in statistical analysis.

Objective probability can be subdivided into Classical Probability and Relative Frequency.

Classical Probability
Classical Probability is based on the assumption that the outcomes of an experiment are equally likely. Using the classical viewpoint, the probability of an event happening is computed by dividing the number of favourable outcomes by the total number of possible outcomes:

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Example 17
The experiment is to observe the "up" face on a six-sided die. What is the probability that a two spot will appear face up?

Solution:
The possible events are:

a one-spot	a four-spot
a two-spot	a five-spot
a three-spot	a six-spot

There is only one "favourable" outcome, a two-spot. All six results for the toss of the die are equally likely. Therefore:

$$\text{Probability of a two-spot} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{1}{6}$$

$$= 0.167$$

If only one of several events can occur at one time, we refer to the events as being mutually exclusive.

Mutually exclusive: The occurrence of any one event means that none of the others can occur at the same time.

In the die-tossing experiment, the six possible outcomes are mutually exclusive events. If a two-spot comes face up on the toss of the die, a five-spot cannot occur at the same time.

If an experiment has a set of events that includes every possible outcome, such as the die-tossing experiment, the set of events is called collectively exhaustive.

Collectively exhaustive: At least one of the events must occur when an experiment is conducted. For the die-tossing experiment, the set of events consists of 1, 2, 3, 4, 5 and 6. The set is collectively exhaustive because it includes all possible outcomes. If the set of events is collectively exhaustive and the events are mutually exclusive, the sum of the probabilities equals 1. For a coin-tossing experiment:

	Probability
Event: Head	0.50
Event: Tail	+ 0.50
Total	1.00

For the classical approach to be applied, the events must have the same chance of occurring. Also, the set of events must be mutually exclusive and collectively exhaustive.

Relative Frequency Concept

Another probability concept is based on relative frequencies. The probability of an event happening in the long run is determined by observing what fraction of the time like events happened in the past. In terms of a formula:

Probability of event happening = Number of times event occurred in past

Total number of observations

Example 18
A study of 882 Marketing graduates at the University of Natal was conducted. This is the experiment. It revealed that 622 out of the 882 were not employed in their major area of study in University. For illustration, a person who majored in accounting is now the marketing manager of an Advertising firm. What is the probability that a particular marketing graduate will be employed in an area other than his or her degree?

Probability of event happening = $\frac{\text{Number of times event occurred in past}}{\text{Total number of observations}}$

$$P(A) = \frac{622}{882} = 0.71$$

Since 622 out of 882 are in a different field of employment from their major in University, we can use this as an estimate of the probability. In other words, based on past experience, the probability is .71 that a marketing graduate will be employed in a field other than his or her degree.

Some Rules of Probability
Rules of Addition
The Complement Rule and
Rules of Multiplication

Rules of Addition
To apply the special rule of addition, the events must be mutually exclusive. Recall that mutually exclusive means that when one event occurs, none of the other events can occur at the same time. As illustrations, if a two-spot comes face up on the roll of a die, none of the other faces (1, 3, 4, 5 or 6) can be face up at the same time.
If tow events A and B are mutually exclusive, the special rule of addition states that the probability of one or the other event's occurring equals the sum of their probabilities. This rule is expressed in the following formula.

Probability of event (A) or event (B) occurring = probability of event (A) + probability of event (B)

$$P(A \text{ or } B) = P(A) + P(B)$$

For three mutually exclusive events designated A, B and C, the rule is written:

P(A or B or C) = P(A) + P(B) + P(C)

Example 19
An automatic machine fills bags with a mixture of small, medium and big chips. Most of the bags contain the correct weight, but because of the slight variation in the size of the chips, a package might be slightly underweight or overweight. A check of 3,000 packages in the past revealed:

Weight	Event	Number of Packages	Probability of occurrence
Underweight	A	100	0.025
Satisfactory	B	2,600	0.900
Overweight	C	300	0.075
		3,000	1.000

What is the probability that a particular package will be either underweight or overweight?

Solution
The outcome “underweight” is the event A. The outcome “overweight” is the event C. Applying the special rule of addition:

P(A or C) = P(A) + P(C)
= .025 = .075
= .10

Note that the events are mutually exclusive, meaning that a packet of chips cannot be underweight, satisfactory, and overweight at the same time. [Also P(A or B or C) = 1.000]

The probability that a bag of chips selected is underweight P(A), plus the probability that it is not an underweight bag, written P(-A) and read “not A” must logically equal 1. This is written:

P(A) + P(-A) = 1

This can be revised to read:

P(A) = 1 – P(-A)

This is referred to as the **complement rule**

Complement Rule
The complement rule is used to determine the probability of an event occurring by subtracting the probability of the event not occurring from 1.

Example 20
Recall that the probability that a bag of chips is underweight is .025 and that the probability of an overweight bag is .075. Use the complement rule to show that the probability of a satisfactory bag is .900.

Solution
The probability that the bag is unsatisfactory equals the probability that the bag is overweight plus the probability that it is underweight.
That is, P(A or C) = P(A) + P(C) = .025 + .075 = .100.
The bag is satisfactory if it is not underweight or overweight, so:
P(B) = 1 – [P(A) + P(C)] = 1 – [.025 + .075] = .900.

The complement rule is very important in the study of probability. Often it is more efficient to calculate the probability of an event happening by determining the probability of it not happening and subtracting the result from 1.

Rules of Multiplication
The special rule of multiplication requires that two events A and B be independent. Two events are independent if the occurrence of one does not alter the probability of the other occurring. So if the events A and B are independent, the occurrence of A does not alter the probability of B occurring.

Independent: The occurrence of one event has no effect on the probability of the occurrence of any other event.

For two independent events A and B, the probability that A and B will both occur is found by multiplying the two probabilities. This is called the special rule of multiplication and is written symbolically as:

P(A and B) = P(A) x P(B)

This rule for combining probabilities presumes that a second outcome is not affected by any previous outcome. To illustrate what is meant by independence of outcomes, suppose two coins are tossed. The outcome of one coin (head or tail) is unaffected by the outcome of the other coin (head or tail).

For three independent events, A, B and C, the special rule of multiplication used to determine the probability that all three events will occur is:

P(A and B and C) = P(A) x P(B) x P(C)

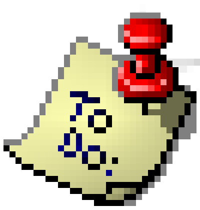
Example 21
Two coins are tossed. What is the probability that both will land tail up?

Solution
The probability of a tail showing face up on one of the coins, written P(A), is one half or .50. The probability that the other coin will land tail up, written P(B), is one half, or .50. Using the above formula, the probability that both will happen is one fourth, or .25, found by:

P(A and B)	=	P(A) x P(B)
	=	½ x ½
	=	¼, or .25

This can be shown by listing all of the possible outcomes. Two tails is only one of the four possible outcomes:

	Tail	&	Tail
Or	Tail	&	Head
Or	Head	&	Tail
Or	Head	&	Head



Exercise 1
This is an exercise in prediction. Take out your set of dice. Before throwing the dice for the first time, write down how many times you think that the number 3 will appear face up if you threw the dice 18 times. Now throw the dice 18 times and note how may times the number 3 actually came face up.

Refer to your notes and work out the probability of 3 coming up in 18 throws. Compare this to the actual number of times that 3 came up.

Now repeat the experiment, but use the number 2.

Interpret and document the results correctly, making sure that the results are clearly communicated

Formative Assessment Activity 1

Please open your Portfolio of evidence (Logbook) and complete the activities for this specific outcome, following the instructions found in your logbook.

Your facilitator will advise you as to the requirements and deadlines for completing and handing in these formative assessments. Write these instructions here:

Unit Standard 9016
Represent analyse and calculate shape and motion in 2 and 3 dimensional space in different contexts

When you have completed this section, you will be able to understand and explain the following:

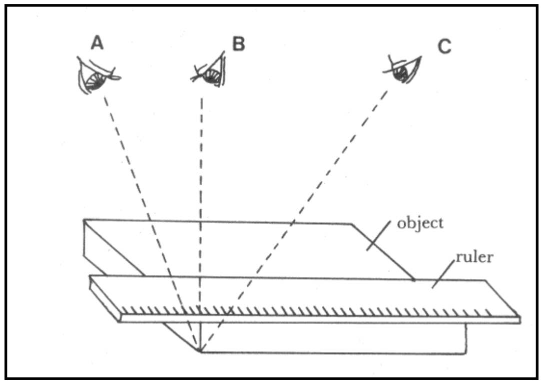
- ✡ Measure, estimate, and calculate physical quantities in practical situations relevant to the adult with increasing responsibilities in life or the workplace
- ✡ Explore analyse and critique, describe and represent, interpret and justify geometrical relationships and conjectures to solve problems in two and three dimensional geometrical situations

<u>Specific Outcome 1</u>
Measure, estimate, and calculate physical quantities in practical situations relevant to the adult.

Physical quantities are things like distance, mass, temperature, speed, acceleration, volume and time. In this section all of these quantities will be described as well as the instruments that are used to measure them. The SI units will be given for the quantities.

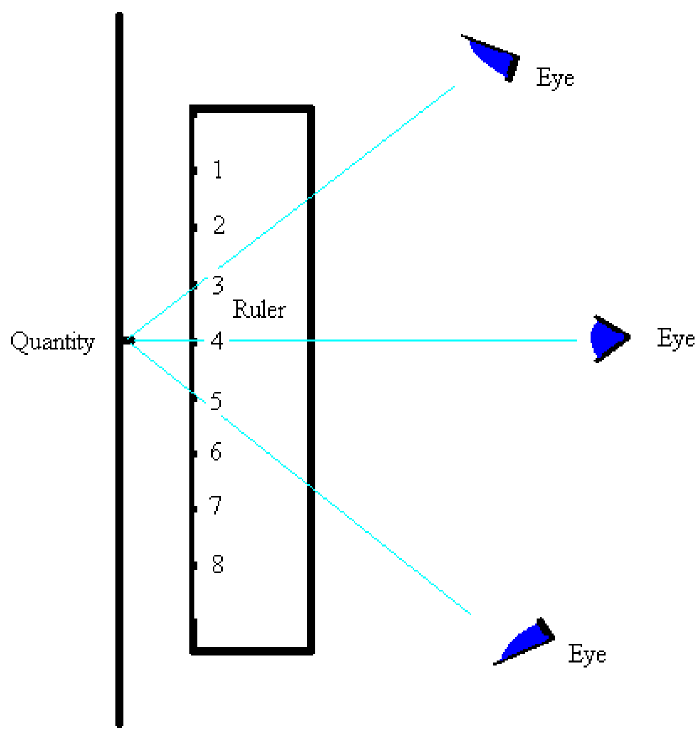
It is very important to avoid parallax error when measuring quantities. Parallax error occurs when the eye, the measuring instrument and the quantity are not in a straight line. An example of parallax error occurs when you are sitting in the front passenger seat of a car. When you look at the speedometer it looks as though the car is moving at a greater speed than it actually is. This is due to parallax error.

Look at the figure below. An accurate measurement can only be made from position B.



The Error of Parallax
Source: New Modern General Science Std 5 NPJ Botha & F.D. Read

Another illustration of the Parallax Error
The three different eye positions in the diagram would yield measurement of 3, 4 and 5. It can be seen that 4 is the correct distance. It is vital to position your eye in the correct position to avoid parallax error when taking readings.



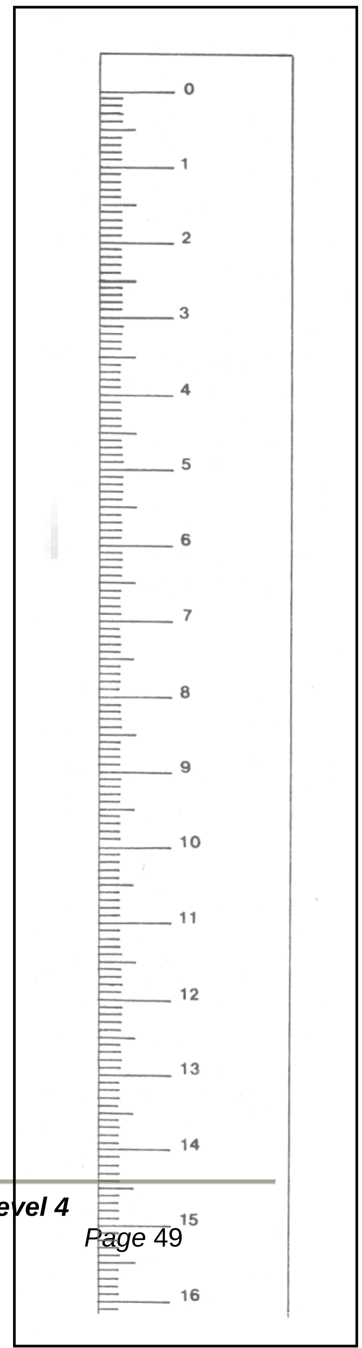
There are three kinds of measurements of the amount of space.

A measurement error that remains constant in magnitude for all observations; a kind of systematic error or what is called a bias error.

Length and Distance

Long ago people used to measure the length of things with their feet, hands and arms. As we all know we all have different size hands, feet and arms therefore using our various body parts to measure length was not such a good idea as the length being measured would vary according to a persons size. Today we have standard measurements. Metres and centimetres, feet and inches are example of these standard measurements. To make exact measurements we use standard measures, which these measurements are the same all over the world.

Length is the measurement of something from one end to the other, we can measure length in centimetres or metres. Small lengths are measured in centimetres while longer lengths are measured in meters.



Instruments we used to measure length are:

- The metre rule
- Measuring tapes
- Callipers

Rulers and Measuring Tapes

The common Ruler and Measuring Tape are suitable for measuring distances from 1 millimeter up to 100 meters. Most rulers are graduated in 1-millimeter increments and have an accuracy of ±0.5 millimeters. Measuring tapes are flexible rulers, and depending on the length of the tape are either graduated in 1-millimeter increments or 10-millimeter increments. If a measuring instrument is graduated to 10-millimeter increments the accuracy would be ±5 millimeters. This means that a reading of 13 millimeters, taken with a ruler graduated in 1millimeter increments, actually could lie anywhere between 12.5 and 13.5 mm. Rulers can only practically be used to measure distances of less than 1 meter. Measuring tapes are useful in situation where surfaces are not straight. When measuring a distance with a tape or a ruler, the zero on the instrument is place at the start of the measurement and the distance is read off the scale. Remember to avoid parallax error. When using a tape, make sure the tape is tight and/or in contact with the surface.

The metre rule

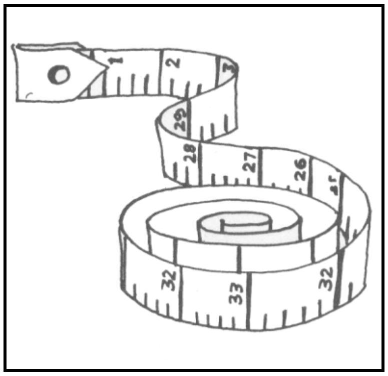
This instrument consists of a wooden rod, exactly one metre or 100 cm long. It is graduated in centimetres and millimetres. For convenience, shorter rods are made.

These are rulers, usually 30cm long, graduated in centimeters and millimetres.

Actual Ruler to scale 1:1
Source: New Modern General Science STD 5 - NPJ Botha & F.D. Read

The Measuring Tape

Dressmakers use a tape marked off in millimetres and centimetres. The tape is usually 1.5m long while builders use measuring tapes of different lengths, from 5m to 50m.



A Dressmakers Measuring Tape
Source: New Modern General Science STD 5 – N.P.J Botha and F.D Read

To measure length we use millimeters, centimeters, metres and kilometres however the main unit of length is the metre.

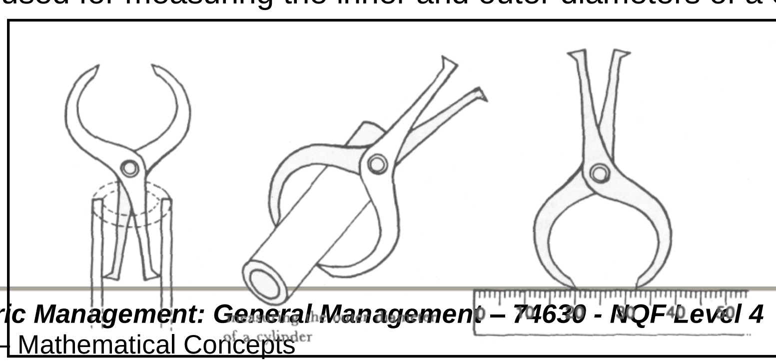
10 millimeters	=	1 centimeter
1000 millimeters/100 centimeters	=	1 metre
1000 metres	=	1 kilometre

The symbols are:

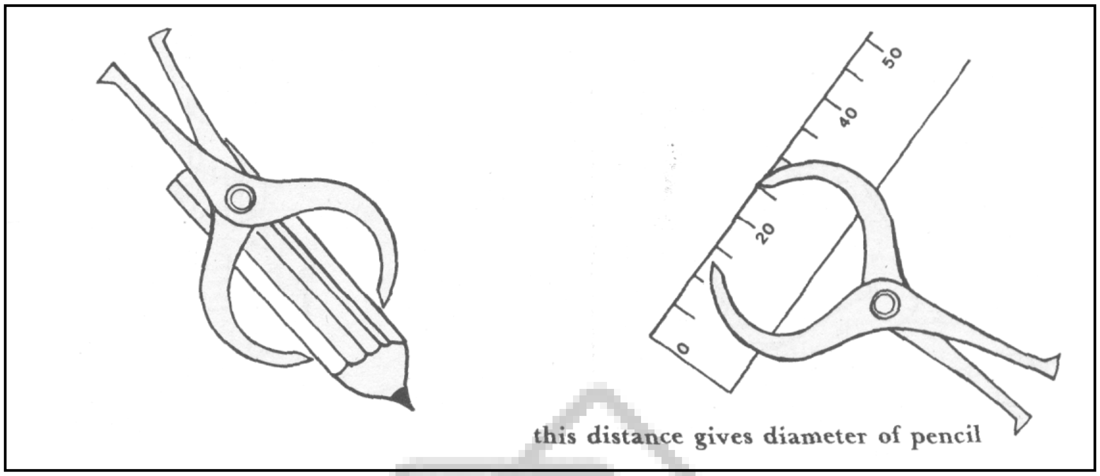
Millimeter	=	mm
Centimeter	=	cm
Metre	=	m
Kilometre	=	km

Calipers

Calipers are used for measuring the diameter of objects which have a round shape, like cylinders, pipes, pencils, nails and wire.
Calipers are used for measuring the inner and outer diameters of a cylinder.



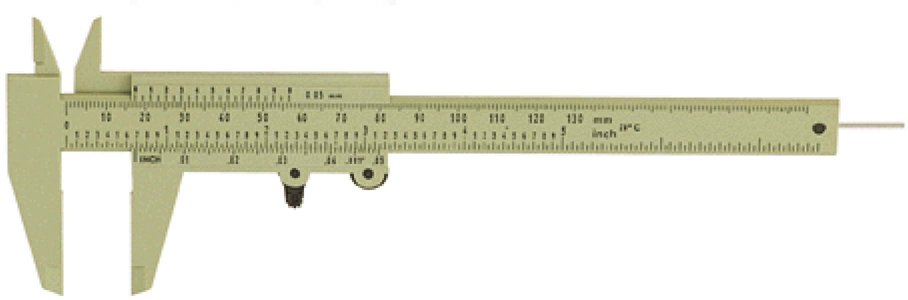
Callipers Measuring the inside and outside diameter of cylinders
Source: New Modern General Science Std 5 - PJ Botha & F.D. Read



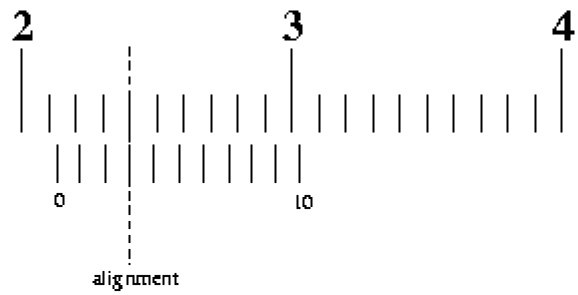
Using Callipers to Measure the Diameter of a Pencil
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read

Vernier Callipers and Micrometer Screw Gauges

A vernier Caliper can measure distances up to a maximum of 30 cm. A vernier caliper can measure down to 0.1 millimeters with an accuracy of $\pm$ 0.05 millimeters. A picture of a vernier caliper is shown below.



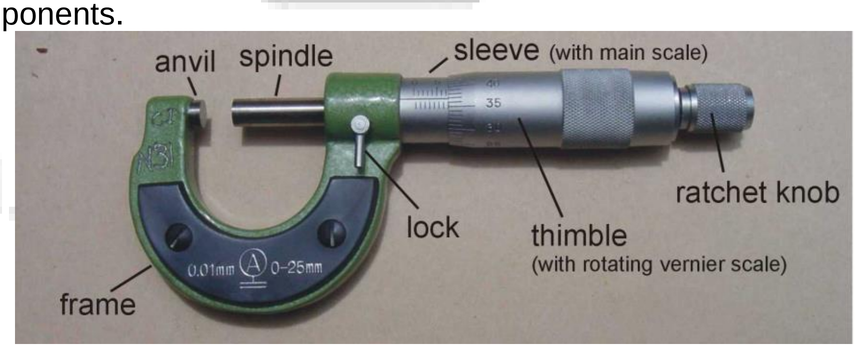
A vernier caliper is very useful for measuring outside and inside diameters of pipes and the dimensions of other small elements. A vernier caliper has a fixed scale and a sliding scale. The numbers on the fixed scale are centimeters; there are ten tick marks on the sliding scale. The left-most tick mark on the sliding scale will let you read from the fixed scale the number of whole millimeters that the jaws are opened. An example is given below.



In the diagram above, the leftmost tick mark on the sliding scale is between 21 mm and 22 mm, so the number of whole millimeters is 21.

Next we find the tenths of millimetres. Notice that the ten tick marks on the sliding scale are the same width as nine ticks marks on the fixed scale. This means that at most one of the tick marks on the sliding scale will align with a tick mark on the fixed scale; the others will miss. The number of the aligned tick mark on the sliding scale tells you the number of tenths of millimetres. In the example above, the 3rd tick mark on the sliding scale is in coincidence with the one above it, so the vernier calliper reading is (21.30 ± 0.05) mm.

Micrometer Screw Gauges can measure distances down to 0.01 ± 0.005 millimeters. They are very useful for measuring extremely thin wires and small spheres and diameters. The picture below shows a micrometer screw gauge with all its components.



Micrometer Screw Gauge

The object to be measured is placed between the anvil and the spindle and the ratchet knob is tightened until it clicks three times. It is very important not to over tighten the instrument because it may get damaged. When taking a reading, the point on the main scale at which the thimble meets the sleeve gives the distance down to 0.1 millimeters. The bottom increment on the main scale gives us the 0.5-millimeter increment. The thimble is then read to get the next 0.01millimeter increment.

Micrometer Screw Gauge Example

The distance in the picture example above is 7.73 ± 0.005 millimetres. See if you get the same answer.

Distance

Distance is the length of space between one point and another

Distance is defined as a length; the SI unit for length is the meter (m). The meter can be divided up into different scales in order to better describe the length of the object being measured. Small distances are often measured in milli-, centi-, or micrometers. Large distances are measured in meters or kilometers. Below are the scales of the units mentioned above as well as the instruments best used to measure the distances.

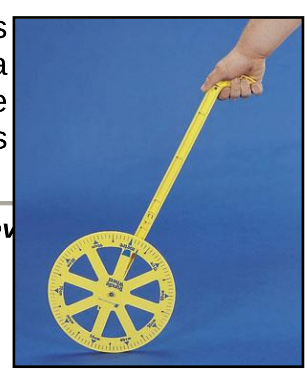
1 micrometer (μm)	=	1×10^{-6} meters	Micrometer Screw Gauge
1 millimeter (mm)	=	1×10^{-3} meters	Small ruler, Vernier caliper
1 centimeter (cm)	=	1×10^{-2} meters	Rulers, Measuring tapes
1 meter (m)	=	1×10^{0} meters	Measuring tapes
1 kilometer (km)	=	1×10^{3} meters	Odometer, Trundle wheel

It is important to choose the correct instrument to measure the quantity in question. If you want to measure the width of a road, it would be stupid to use a ruler because it won't be long enough. If you need to measure a curved surface one would have to use a flexible tape to conform to the shape of eh surface. Micrometer Screw Gauges and Vernier Caliper should be used in situations that require sub-millimeter accuracy.

Instruments we use to measure large distances are:

- Trundle wheel
- Odometer

When large distances need to be measured in situations where a measuring tape will be ineffective, an odometer or a trundle wheel can be used. They both work on the same principal. The circumference of a wheel is calculated, it is



rolled along the ground for a certain distance and the number of revolutions done over the distance is counted. The distance travelled is the circumference multiplied by the number of revolutions. Some odometers and Trundle wheels have been calibrated through mechanical gearing so that a distance reading is given automatically. Your odometer in your car is an example of this. When you look at your dashboard, you see that you have done 541 km, not 344440 wheel revolutions (Based on a 50 cm diameter tyre). A picture of a small trundle wheel can be seen below.

A Trundle Wheel

Geometry

The word geometry is Greek for geos - meaning earth and metron - meaning measure.

Geometry is the study of angles and triangles, perimeter, area and volume.

It differs from algebra in that one develops a logical structure where mathematical relationships are proved and applied.

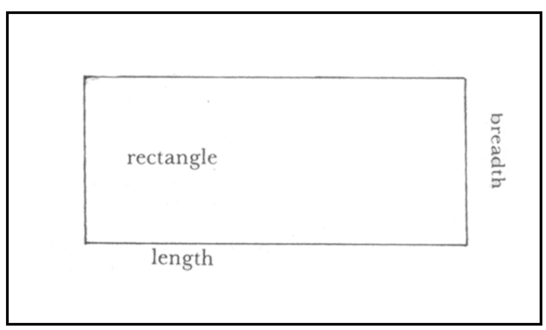
In geometry, a plane is a flat two-dimensional "surface" (similar to a sheet of paper, but with no thickness and no finite length or width). A plane is defined by three points, each of which forms a line with the other two points within the plane. In three dimensions, the simplest version of a plane would include all of the points with any x and y value that contain the same value for z.
A plane is a flat surface or a 2-dimensional object, stretching to infinity in all directions.

Solid geometry - The branch of mathematics that deals with three-dimensional figures and surfaces.

Area
Area is the size of a surface which is commonly measured in square units.

The formula for finding the area of a **rectangle**:

Area = length x breadth

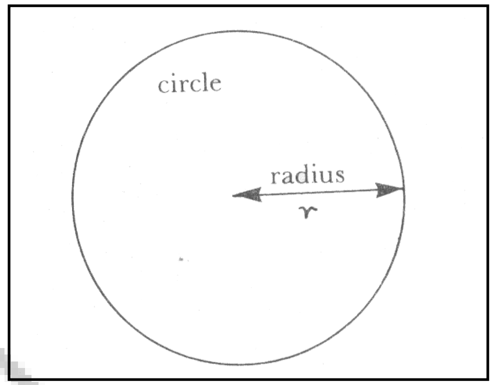


Finding the Area of a Rectangle
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read

The formula for finding the area of a **circle**:

Area = πr^2 or

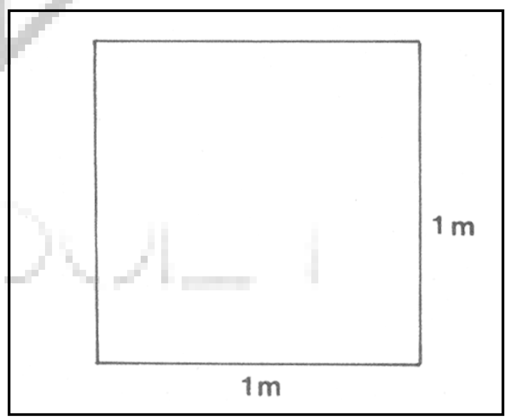
Area = $\frac{22}{7} \times r^2$ (r is the radius of the circle)



Finding the Area of a Circle
Source: New Modern General Science
Std 5 – N.P.J Botha and F.D Read

The formula for finding the area of a **square**:

Area = the length of one side **x** by itself, since all sides are the same length
= length x length



Finding the area of a Square
Source: New Modern General Science
Std 5 – N.P.J Botha and F.D Read

To measure area, we use square centimetres and square metres. For large areas we use hectares.

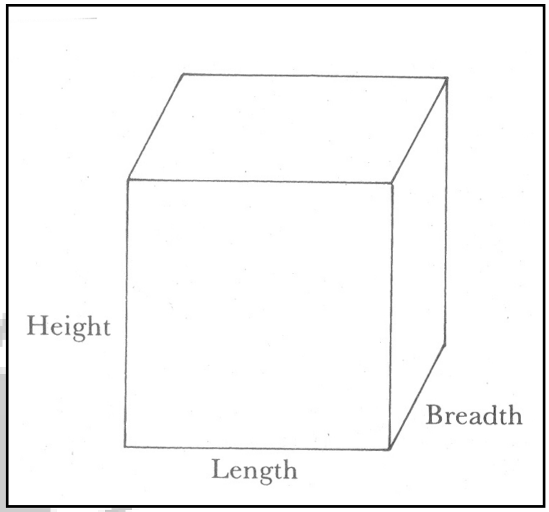
100 square centimetres (cm^2)	=	1 square decimetre
10 000 square centimetres (cm^2)	=	1 square metre (m^2)
10 000 square metres (m^2)	=	1 hectare (Ha)

Volume

Matter occupies space. The amount of space that a substance occupies is known as volume in other words volume is the amount of space taken up by a shape. A pen and pencil take up less space in a bag than books do.

The formula used for calculating the volume of a **rectangular** object:

length x breadth x height
(l x b x h)



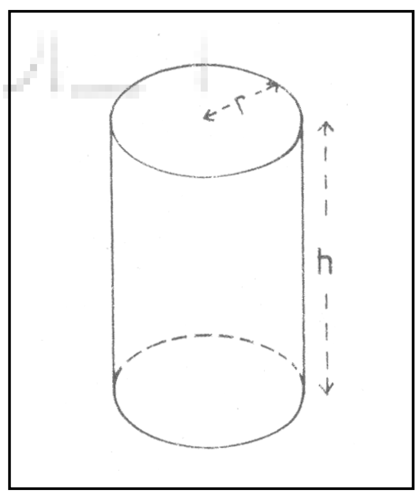
Volume of Rectangular Block
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read

The formula used for calculating the volume of a cylinder-shaped object:

Volume = base area x height

$= \pi r^2 \times h$

$= \frac{22}{7} \times r^2 \times \text{height}$



Volume of Cylinder
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read

Instruments used for measuring Volume:

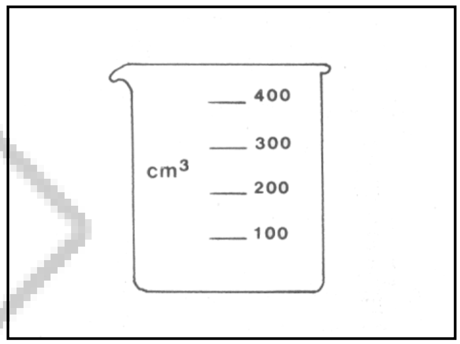
- The measuring beaker
- The measuring cylinder
- The burette
- The pipette and

- The syringe

These instruments are made of glass or plastic. They have measuring scales marked on the outside.

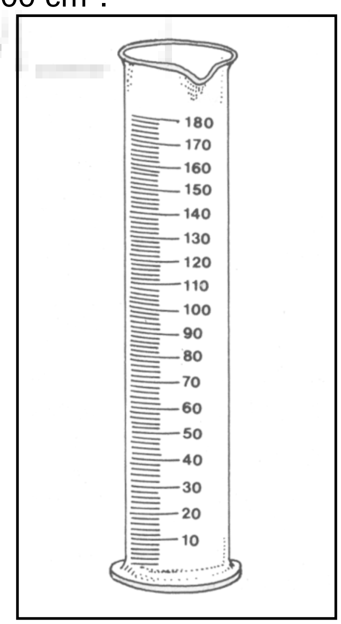
The Measuring Beaker
A measuring beaker is not a very accurate instrument for measuring volume. The scale on the side tells the volume of liquid it contains. The common sizes of measuring beakers range between 100 cm^3 and 1 000 cm^3.

The Measuring Beaker
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



The Measuring Cylinder
A measuring cylinder is the most often used volume measuring instrument. It is more accurate than the beaker. The scale on the side tells the volume of liquid that has been poured into it in cubic centimetres. The largest division is at the top. The common sizes are 250 cm^3, 500 cm^3 and 1 000 cm^3.

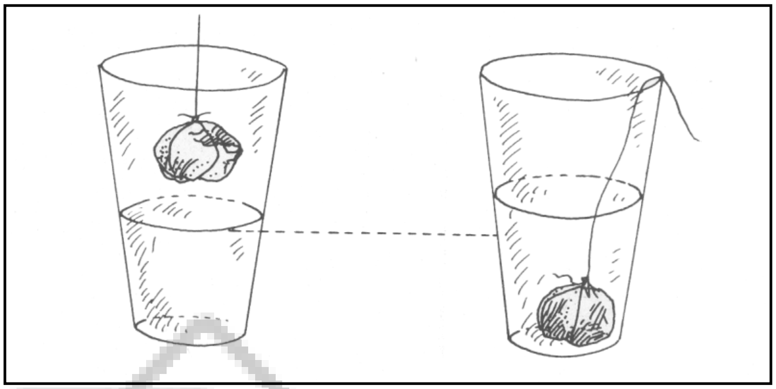
The Measuring Cylinder
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



Volume and displacement of a liquid

When a stone is dropped into a glass of water the level of the water rises. The stone moves some of the water away. The stone has displaced the water. The volume of water that is displaced is equal to the volume of the stone. For example if 15 cubic centimetres of water is displaced, it means the volume of the stone is 15 cubic centimetres.

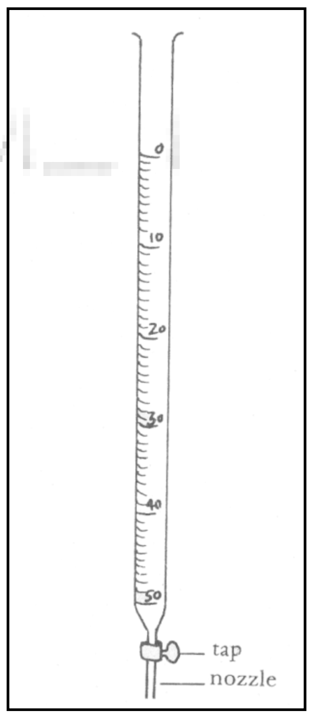
Finding the Volume of an irregular-shaped object
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



The Burette

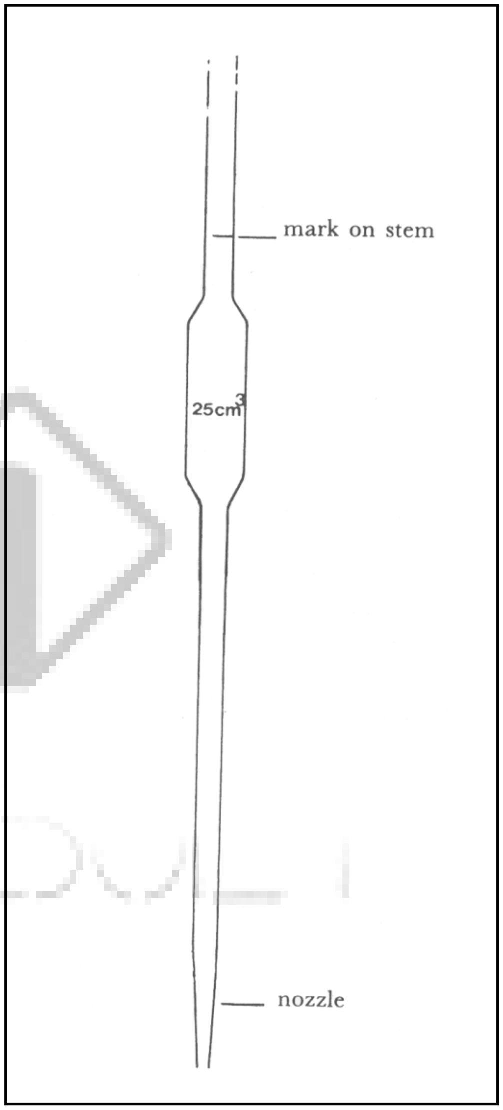
A burette is used for very accurate measurement of the volume of liquids. The scale on the side of the burette shows how much liquid flows out when the tap is opened. The most common size of burette is 50 cm^3. The zero mark is at the top and fifty at the bottom. When filled to the zero mark, the burette indicates that no liquid has flowed out. For example a reading of, say thirty means that, thirty cubic centimetres of liquid has flowed out.

The Burette
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



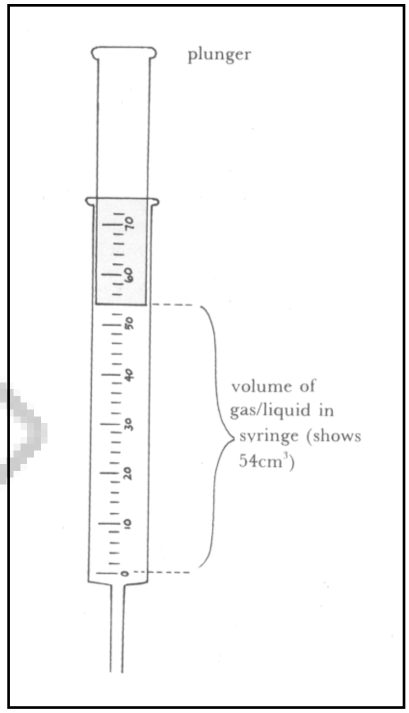
The Pipette
A pipette is used for transferring an amount of liquid from one container to another. A pipette can only hold a fixed volume of liquid. It contains this volume when filled up to the mark on the stem. The common pipette sizes are: 20 cm^3, 25 cm^3 and 50 cm^3

The Pipette
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



The Syringe
Another instrument used for measuring volume is the syringe. Most of us have experienced the doctor's syringe used for injecting a fixed volume of liquid into the blood stream. The outer sleeve of the syringe is marked with a scale. The position of the plunger shows how much liquid or gas is contained in the syringe. The syringe indicates directly the volume of a gas or liquid it contains. In other words a syringe gives a direct measurement of volume.

The Syringe
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



Weight
Weight is how heavy something is.
The weight of an object depends on both its size and what it is made of.
For example a tiny piece of lead weighs the same as a huge bag of feathers.
A scale is used for weighing things
When we want to find out how much something weighs, we use scales.
Small weights are measured in grams (g) while heavier weights are measured in kilograms (kg).
For example when we want to find out how much we weigh we stand on a scale, which gives weight in kg's and when we want to weigh food we use a kitchen scale which measures in grams and no more than 5 kg's.

Mass and Weight by Volume
Another method of determining the mass or weight of an object is to measure the volume of that object. Once the volume is known, it is multiplied by the object's density to determine its weight. The formula it given below:

$$\text{Mass (kg)} = \text{Volume (m}^3) \times \text{Density (m}^3\text{/kg)}$$

In order to find its volume, a number of techniques can be used. If the quantity to be measured is a liquid, then a measuring cylinder or jug can be used. These are containers with a scale on the side. The liquid is pored into the vessel and the volume is measured from the scale. It is important to remember than when reading the volume off the scale not to include the height of the meniscus in the reading. The meniscus is formed because of the surface tension between the sides of the beaker and the water.

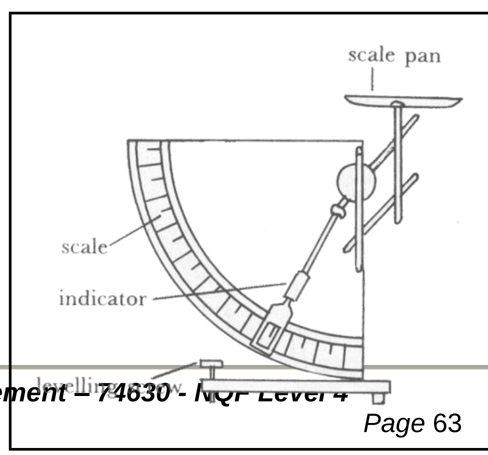
To determine the volume of solid regular objects, their volumes can be determined by directly measuring the dimensions and calculating the volume. If the object is irregular the following method can be used:

> Fill a measuring beaker or jug with a quantity of liquid. Measure the volume. Then immerse the irregular object in the liquid so that it is totally submerged. Measure the new volume. The difference between the first and second volume is the volume of the object.

The volume of an object can also be determined if the mass and density of the object is known. Just simply rearrange the equation given at the top of the page and solve for the volume.

Mass
Mass is the amount of matter in an object, the main unit for measuring mass is in kilograms (kg). Comparing the mass of different material can be done using a balancing scale. The instrument for measuring mass is called a massmeter or balance. There are various types of balances. Electrical massmeters are capable of measuring the mass of very small objects accurately.

The lever-arm balance
This type is known as a direct-reading balance. When an object is placed on the scale pan the indicator points to the mass value on the scale. Before using the balance the indicator must be set pointing to the zero (0) mark on the scale. This is done by means of the levelling screw.

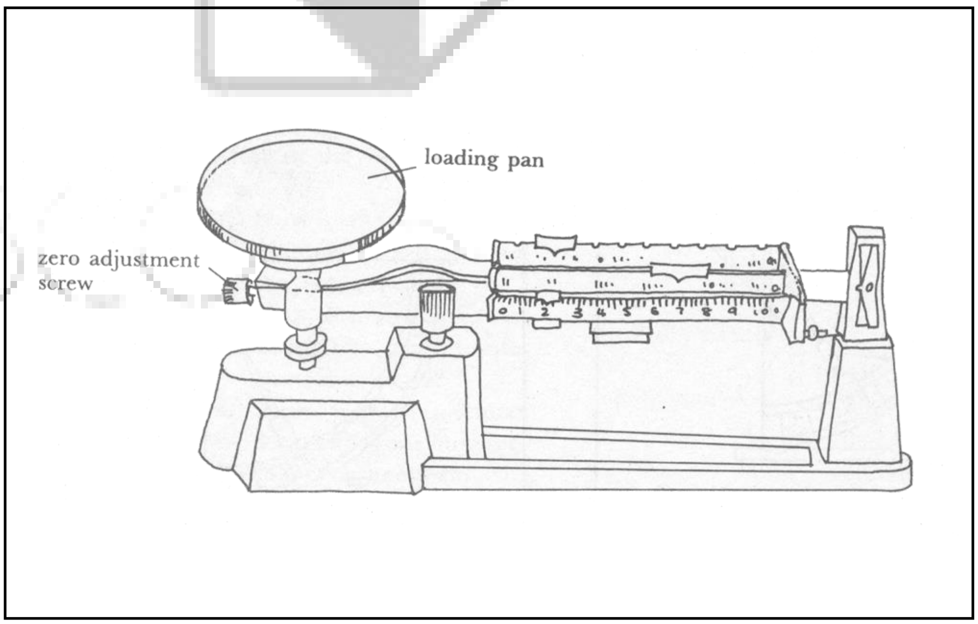


The Lever-arm Balance
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read

Mass Balances can also be used to measure the mass of items and objects. For example if you were wanting to measure out 1kg of sugar, you would place 1kg of weights on the one side of the balance and fill the scale up with sugar until the two sides of the scale balance. This method was used more in the past and is not currently used much. The Triple Beam scale is an example of the mass balance principle. An object is placed on the scale and the three different masses on each of the three beams are moved until the mass of the object and the cumulative masses on the three beams are equal. The sum of the three beam weights is then the mass of the object.

The triple-beam balance
The object to be massed is placed on the loading pan. The set mass pieces are moved along the three beams, until the pointer is at the zero mark. Each mass piece points to a number on its beam. These numbers are added together to get the mass of the object.

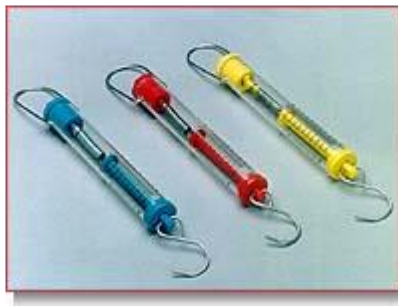
The Triple-beam Balance Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



Triple Beam Balance

A spring balance

A spring balance consists of a spring with a hook and a ruler that has been calibrated for specific weight. The manufacturers worked out that when a 1 kg weight was hung from that particular spring, it deflected 1cm. Therefore, they made a ruler scale where 1 kg represented 1 cm and attached to the spring. So when the spring deflected 3 cm, you knew the object weighed 3 kg. When you look at a spring scale it doesn't seem that simple, but at least you now understand the principle behind it.



Spring Balances

There are also very accurate digital scales that can be used to measure weights of less than one gram. If the situation requires, it is worth noting that such accuracy is available, but as usual, these types of scales are normally far more expensive.

Mass and Weight

It is important to distinguish between mass and weight. The mass of an object is a fundamental measure of the amount of matter in the object. The weight of an object is the force of gravity acting on that object. Mass is difficult to quantify in terms of something else. The SI unit for mass is the Kilogram (kg). The weight of a body at a point is defined as the mass of the body multiplied by the force of gravity at that point. The SI unit for weight, which is a force, is the Newton (N).

An example of weight and mass is if you had to weigh yourself in Durban, and then on the moon. The force of gravity on earth is 9.86 m^2/s. on the moon it is approximately one sixth of that, i.e. 1.64 m^2/s. If your mass on earth was 70 kg. Your would then weigh:

Weight in Durban: 70kg x 9.86 m^2/s = 690.2 N

Weight on the Moon: 70kg x 1.64 m^2/s = 115.0 N

As you can see, your weight changes because the force of gravity in the two places is different. However, you still have the same mass.

Temperature

The SI unit for temperature is degrees Kelvin (K).

Temperature measures the degree of hotness or coldness of an object. A scale of 0 to 100 is usually used to measure heat. Hot objects have a high temperature while cold object have a low temperature. The two units used are degrees Celsius ($°C$) and or degrees Fahrenheit ($°F$). On a Celsius' scale water freezes at $0°C$ and boils at $100°C$ and on Fahrenheit's scale water freezes at $32°F$ and boils at $212°F$.

Thermometers either have to be read manually or some have a digital readout that gives you the temperature.

The instrument used to measure temperature is a thermometer.

The following equations are used to convert temperature between °C and °F.

$$°F = (°C \times 1.8) + 32$$

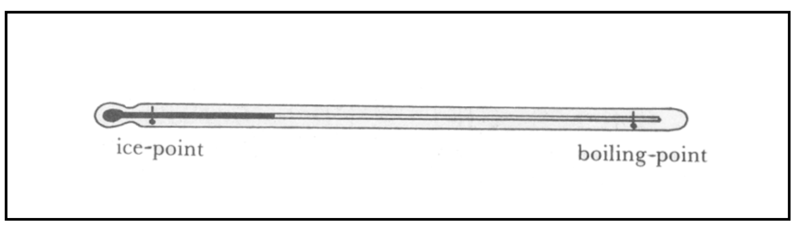
$$°C = (°F - 32) / 1.8$$

It is useful to know the conversion in order to quickly change from the one measuring system to the other. The most commonly used temperature scale used in South Africa is Degrees Celsius.

The Thermometer

There are various types of thermometers. The most common type is the mercury thermometer. This thermometer consists of a glass tube. There is a bulb of thin glass at one end, filled with mercury. The fine hollow down the middle is called the bore. It has the same width along the entire length. The other end of the thermometer is sealed.

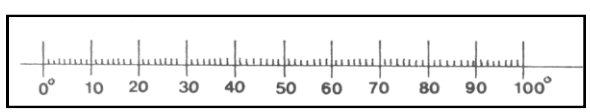
The Thermometer
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



Equal divisions, called degrees, are marked along the stem of the thermometer. The letter $°C$ at the top indicates that the thermometer has been marked in Celsius degrees.

The lower mark is $0°C$

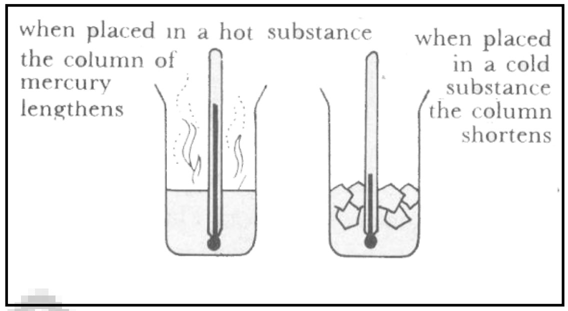
The upper mark is $100°C$



A Scale for Measuring Temperature
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read

On a warm day the mercury or coloured alcohol gets hotter. It climbs higher up the tube. The higher the temperature the higher the mercury or alcohol climbs. On a cool day the mercury or coloured alcohol cools. The lower the temperature the lower the mercury or coloured alcohol moves down the tube.

The Mercury Type Thermometer
Source: New Modern General Science Std 5 – N.P.J Botha and F.D Read



Time, Speed and Acceleration
The SI unit for time is seconds (s). Time can be measured with watches and stopwatches. Time can be combined with distance to yield velocity. The formula for calculating velocity given time and distance is given below:

$$\text{Velocity (m/s)} = \frac{\text{Distance (m)}}{\text{Time (s)}}$$

An example of this formula is given.

<u>Example:</u> It took 5 hours and 10 minutes to travel from Durban to Johannesburg. The distance from Durban to Johannesburg is 650 kilometers. What was the average velocity in meters per second (m/s)

The first point to note is that we have kilometers and hours and we need an answer in meters and seconds. We know that there are 1000 meters in every kilometer. We also know that there are 60 minutes in an hour and 60 seconds in a minute. Therefore, doing the conversions:

Distance = 650 (km) x 1000 (m/km)
= 650 000 meters

Time = (5 hrs x (60 x 60)) + (10 mins x 60)
= 18 600 s

$$\therefore \quad \text{Velocity (m/s)} = \frac{\text{Distance (m)}}{\text{Time (s)}}$$

$$= \frac{650\,000}{18\,600}$$

$$= \underline{\mathbf{34.94\ m/s}}$$

We could have done the calculation in km and hrs to yield **125.81 km/hr**

It is important to remember that acceleration is defined as the rate of change of velocity. The units for acceleration are m^2/s. acceleration is therefore not the same as velocity. Acceleration can be positive or negative. Negative acceleration is sometimes refers to as deceleration.

Speed and Acceleration
Speed is how fast something is moving or is the rate of movement, a measure of distance travelled in a unit of time. In other words speed describes how far an object travels in a period of time. A speedometer indicates how fast a car is moving. We use speedometers to measure speed.

Acceleration is how much the speed increases in a period of time or is the rate of change of speed, which is usually measured as feet or meters per second. A decrease in speed is called deceleration.

Points to remember when measuring

- Avoid parallax error by keeping your eye, the instrument and the point to be measured in a straight line.
- Don’t include the meniscus when measuring the volume of liquids.
- Measure to the last step of the instrument. For example, if using a micrometer screw, your answer should be down to (0.01 $\pm$ 0.005) mm, a normal ruler can only measure to (1 $\pm$ 0.5) mm.
- The unit an accuracy of the measuring instrument is always given somewhere on the instrument, so you should always know what increment you are measuring.
- Use the SI convention of units.

Standard International System of Units
The Standard International System of Units is abbreviated as the SI system. This system of measurement is continually replacing the traditional measuring systems, such as the imperial unit system, that are often used in science and the

practical context. They are also becoming more universally accepted in commerce and industry.

The table below shows the SI system of units.

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Luminous Intensity	Candela	cd
Amount of Substance	Mole	mol

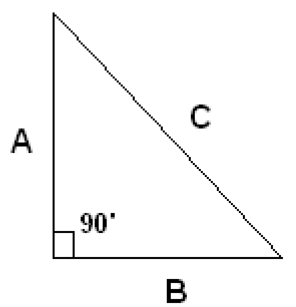
What is Calibration of the Instruments?
Calibration of the measuring instrument is the process in which the readings obtained from the instrument are compared with the sub-standards in the laboratory at several points along the scale of the instrument. As per the results obtained from the readings obtained of the instrument and the sub-standards, the curve is plotted. If the instrument is accurate there will be matching of the scales of the instrument and the sub-standard. If there is deviation of the measured value from the instrument against the standard value, the instrument is calibrated to give the correct values.

All the new instruments have to be calibrated against some standard in the very beginning. For the new instrument the scale is marked as per the sub-standards available in the laboratories, which are meant especially for this purpose. After continuous use of the instrument for long periods of time, sometimes it loses its calibration or the scale gets distorted, in such cases the instrument can be calibrated again if it is in good reusable condition.

Even if the instruments in the factory are working in the good condition, it is always advisable to calibrate them from time-to-time to avoid wrong readings of highly critical parameters. This is very important especially in the companies where very high precision jobs are manufactured with high accuracy.

The Theorem of Pythagoras

Pythagoras's Theorem is often used to calculate heights and distances. The theorem applies to right-angle triangles only. The theorem is described in the diagram below and practical applications of the theory are given after.



Pythagoras Theorem:

$$A^2 + B^2 = C^2$$

For a right angled triangle with two sides **A** and **B,** and with Hypotenuse **C**

Example:

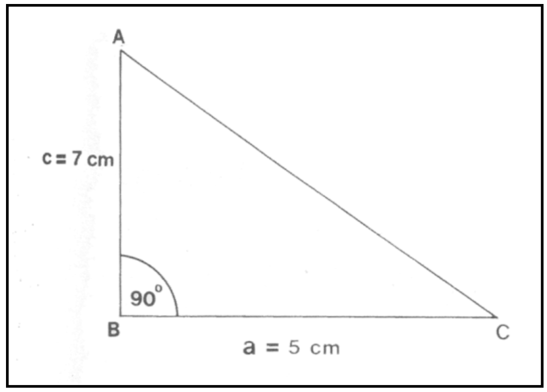
A builder wants to know the diagonal distance across a perfectly rectangular room. One side of the room is 3 meters long (A); an adjacent wall is 4 meters long (B). The angle between them is 90º. To calculate the diagonal (C), use the

Pythagoras Theorem:
$C^2 = A^2 + B^2$
Therefore:

$$C = \sqrt{A^2 + B^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ meters}$$

The diagonal across the room is therefore 5 meters

Example: ΔABC has a = 5cm, c = 7cm and $\angle B = 90°$.
Calculate b.



Source: New Mathematics Std 6 – Dr JL van Dyk and PJA du Toit

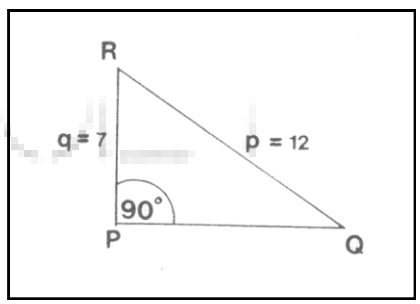
$b^2 = a^2 + c^2$

$b^2 = 5^2 + 7^2\ cm^2$
$= 25 + 49\ cm^2$
$= 74\ cm^2$

$\therefore$ b = $\sqrt{74cm}$

Leave the answer in this form when the square root cannot be easily calculated.

Example: ΔPQR has p = 12m, q = 7m and the right angle at P.
Calculate PQ.



Source: New Mathematics Std 6 – Dr JL van Dyk and PJA du Toit

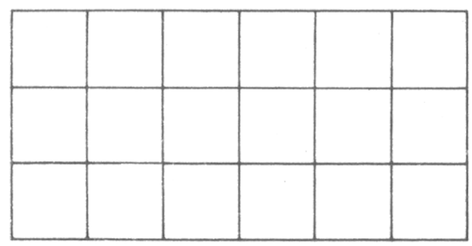
$p^2 = q^2 + r^2$

$\therefore\ 12^2 = 7^2 + r^2$
or $r^2 = 12^2 - 7^2\ m^2$
$= 144 – 49\ m^2$
$= 95m^2$
$\therefore\ r^2 = \sqrt{95m}$

A. Calculating Surface Areas and Volumes using Right Prisms

1. The area of a rectangle
   A right prism
   The area of a right prism on a square base

   a. The area of a rectangle



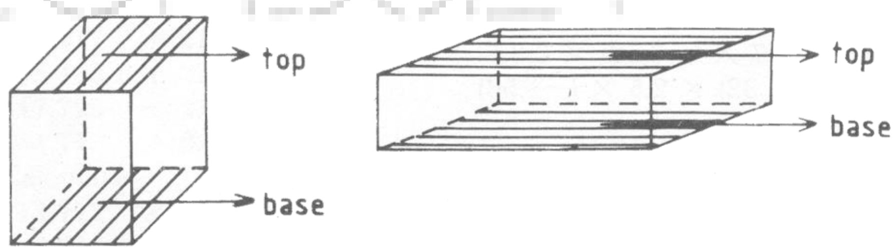
Source: Mathematics Explained Std 8 – JJ de Wet

The rectangle in the diagram above is covered with square metres (drawn to scale).
Count the number of square metres placed on the square. How many are there?
The answer is 18. Hence we can say that the area of the rectangle is 18m^2.
There are 6 squares in a row and there are 3 rows.

The area of the rectangle $= (6 + 6 + 6)$ m^2
$= 6 \times 3$ m^2
$= L \times B$

(If L and B represent the length and breadth of the rectangle respectively)

   b. A right prism



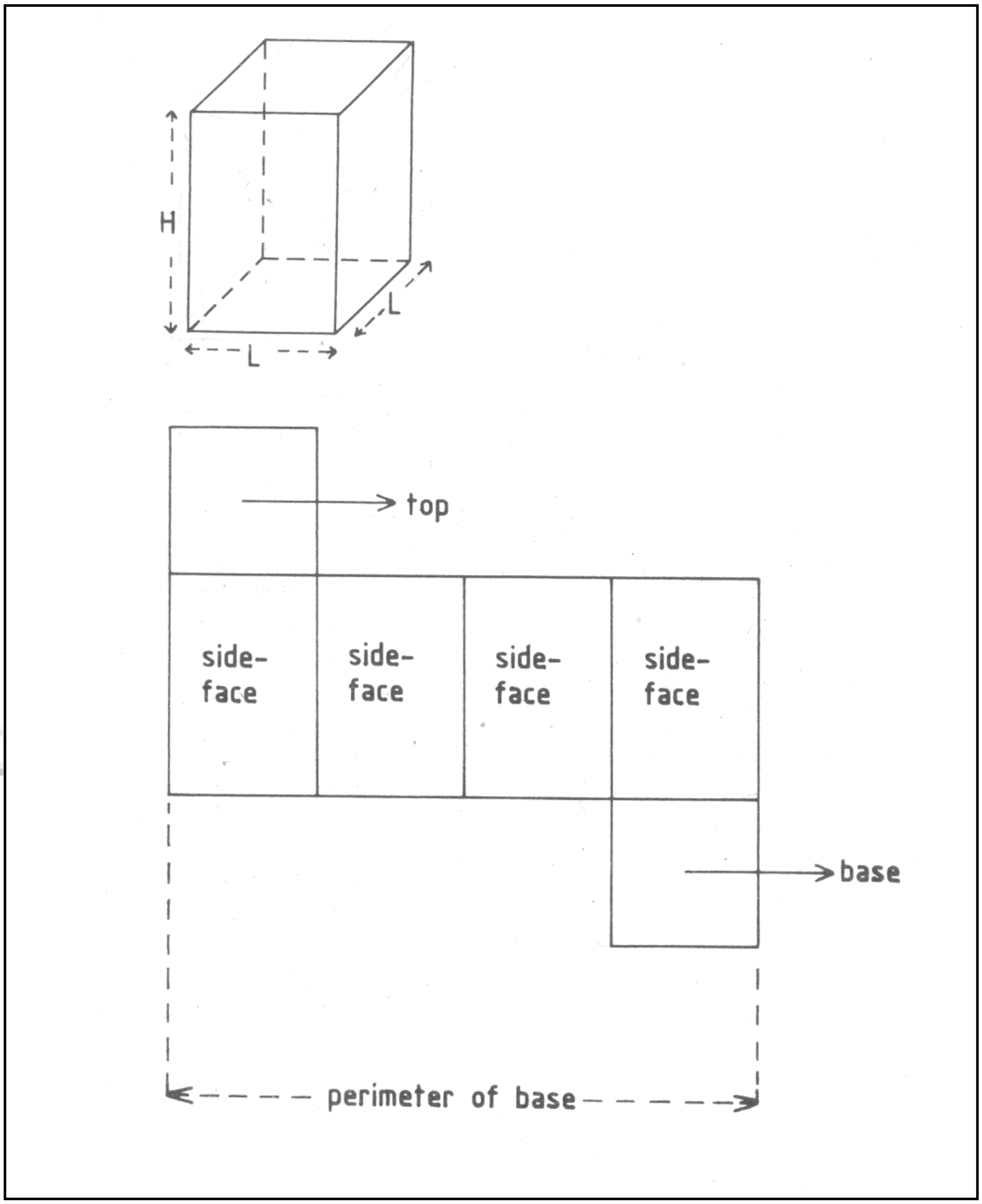
Source: Mathematics Explained Std 8 – JJ de Wet

The bottom face of a prism is called the base of the prism.
The upper face of a prism is called the top of the prism.
The bases and tops of the prisms drawn in the previous diagrams are coloured.
If the side faces are perpendicular to the base and the top, the prism is called a right prism.
Note that the top and the base of such a prism are congruent. A match box is a good example of a right prism.

c. The area of a right prism on a square base

In the following diagram a box in the form of a right prism on a square base and its unfolding are shown:

Source: Mathematics Explained Std 8 – JJ de Wet



Suppose that the length of a side of the base is represented by L and the height of the prism by H.

Note that the areas of the side faces are equal.

The area of a side face = L H

Thus the area of the 4 side faces = 4 L H

Note that the area of the base and the top are equal

The area of the base = L x L = L^2

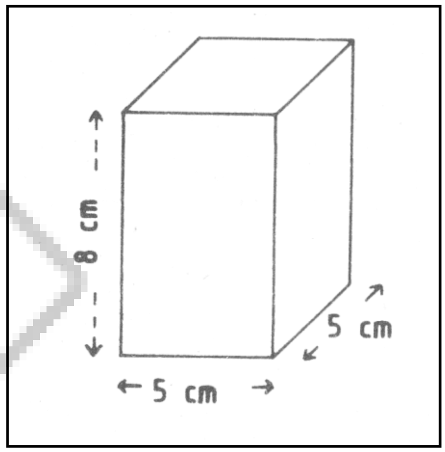
∴ The area of he base and the top = 2 L^2

The surface area of a right prism on a square base

= Area of the side face + Area of the top and the base

= 4 L H + 2 L^2

Example: Calculate the surface area of the following right prism with a square base.



Source: Mathematics Explained Std 8 – JJ de Wet

The surface area of the right prism

= 4 LH + $2L^2$

= (4 x 5 x 8 x 2 x 5^2) cm^2 since L = 5cm and H = 8 cm

= (160 + 2 x 25) cm^2 since 4 x 5 x 8 = 160 and 5^2 = 25

= (160 + 50) cm^2

= 210 cm^2

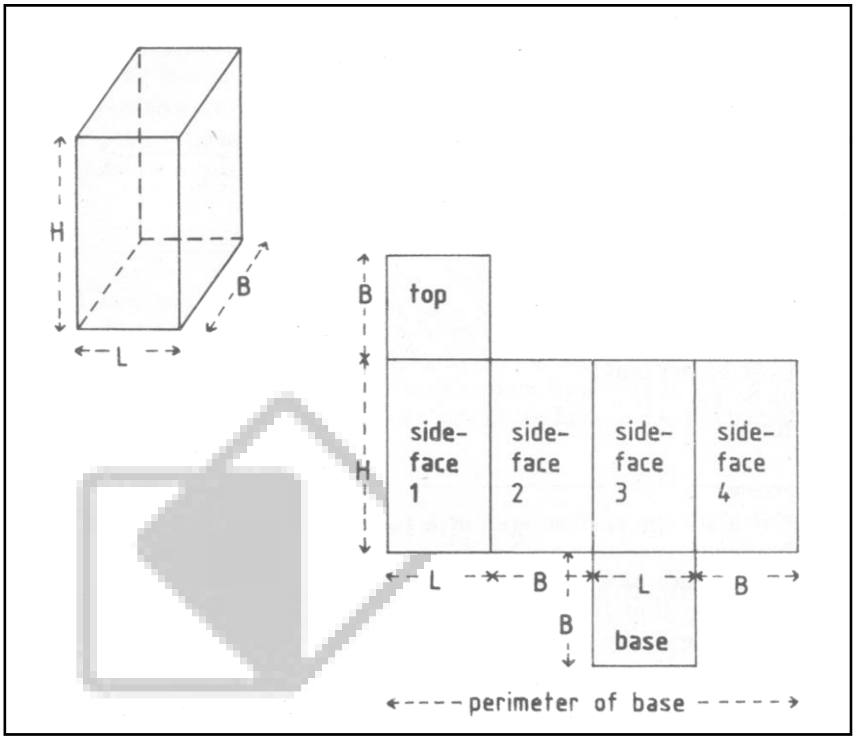
Remember

1. The area of a rectangle = length x breadth (A = L x B)
2. The area of a square = length x length (A = L^2)
3. The area of a right prism on a square base = 4 LH + $2L^2$

B. The Surface Area of a Right Prism with a rectangular base

In the following diagram a box in the form of a right prism on a rectangular base and its unfolding are shown:

Source: Mathematics Explained Std 8 – JJ de Wet



Note that the areas of the side faces indicated by 1 and 3 are equal.

The area of the side face indicated by 1 = LH
$\therefore$ The areas of the side faces indicated by 1 and 3 = 2 LH
The area of the side face indicated by 2 = BH
$\therefore$ The areas of the side faces indicated by 2 and 4 = 2 BH

Note that the area of the top and the base are equal
The area of the base = LB
The area of the top and the base = 2 LB
$\therefore$ The surface area of a right prism with a rectangular base
= 2 LH + 2 BH + 2 LB
= 2(LH + BH + LB)

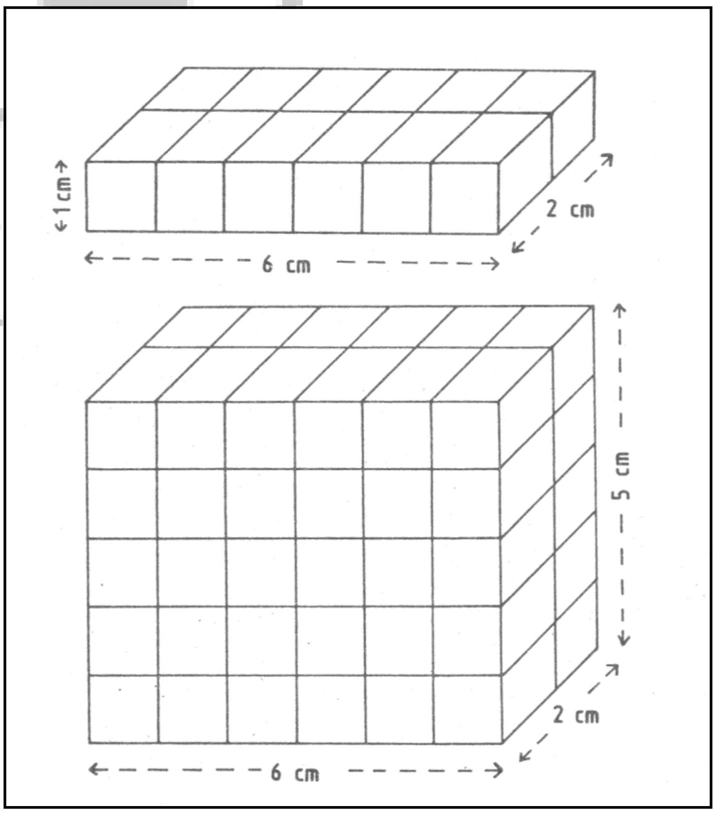
Example:

Find the surface area of a right prism with a length of 4,5m; a breadth of 5,2m and a height of 8,4m.

The surface area of the right prism
= 2(LH + BH + LB)
= 2(4.5 x 8.4 + 5.2 x 8.4 + 4.5 x 5.2) m^2
= 2(37.8 + 43.68 + 23.4) m^2
= 2(104.88) m^2
= 209.76 m^2

Remember
The surface area of a right prism = 2(LH + BH + LB) where L represents the length of the base of the prism, B the width of the base and H the height of the base.

C The Volume of a Right Prism with a Rectangular Base



Source: Mathematics Explained Std 8 – JJ de Wet

The rectangular prism was built by putting five of the layers on to each other. How many cubic centimetres were put together to make the layer?

The answer is 12. Instead of counting, we can calculate the number of cubes by multiplying the length by the width of the base, i.e. 6 x 2. How many cubic centimetres were put together to make the rectangular prism?
The answer is 60.

Instead of counting we can calculate the number of cubes by multiplying 12 by the number of layers, i.e. 12 x 5 or 6 x 2 x 5
The volume of the rectangular prism is therefore 60 cm^3
We note that the volume of the prism can be calculated by multiplying the length by the width, by the height. If the volume is denoted by V we can say:
V = LBH

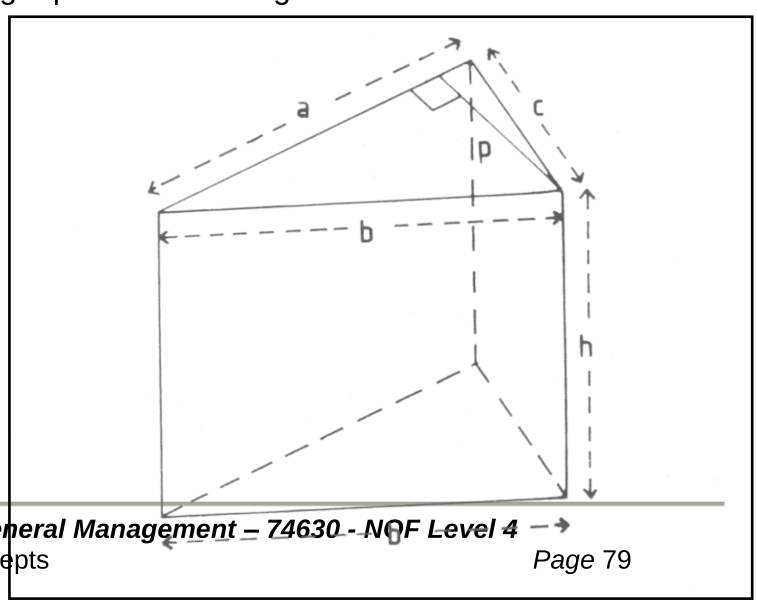
= Area of base x height since LB represents the area of the base.
If the right prism has a square base we can say that L = B.
Hence V = LLB [Substitute B by L]
= L^2 B [Since L x L = L^2]

Example
A match box is 50 mm long, 30 mm wide and 20 mm high. Calculate the volume of the box in cubic centimetres.
First change the millimetres to centimetres
50mm = 5 cm; 30mm = 3cm and 20mm = 2cm
Volume = L x B x H
= 5 x 3 x 2 cm^3
= 30 cm^3

Remember
The volume of a right prism with a rectangular base is given by the product of the area of the base and the height.
Volume = area of base x height
V = L x B x H

D. The Surface Area and Volume of a Right Prism on a Triangular Base

1. The surface area of a right prism on a triangular base



Source: Mathematics Explained Std 8 – JJ de Wet

The area of base triangle = ½ x base x height
= ½ x a x p

= ½ ap

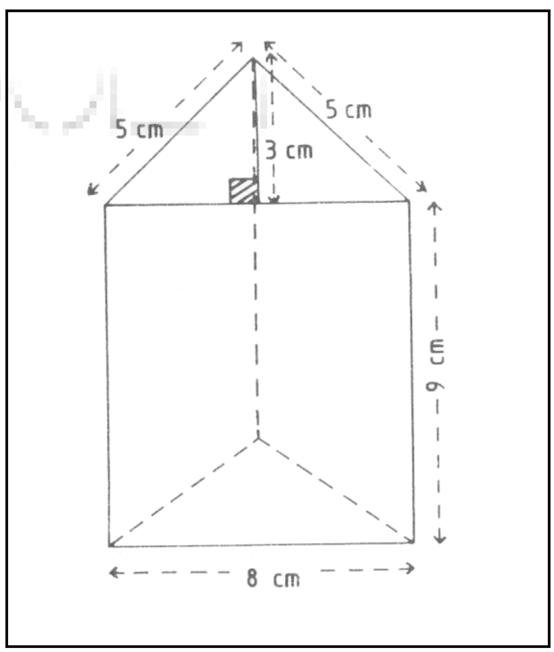
The area of the top triangle= ½ ap

The total area of the base and top = 2 x ½ ap
= ap

The area of the side face with length a units = ah
The area of the side face with length b units = bh
The area of the side face with length c units = ch
The total area of the side faces = ah + bh + ch
∴ The surface area of the right prism with a triangular base
= ap + ah + bh + ch

2. The Volume of a right prism on a triangular base

Consider the previous diagram.
With a more advanced knowledge of mathematics it can be proved that the volume of any right prism is always given by the product of the area of the base and the height of the prism.
The volume of a right prism on a triangular base
= Area of base x Height
= ½ ap x h
= ½ aph



Source: Mathematics Explained Std 8 – JJ de Wet

Example:
Calculate the surface area and the volume of the solid represented in the accompanying diagram.

Area of base = ½ BH
= ½ x 8 x 3 cm^2
= 12 cm^2

Area of top and base = 24 cm^2

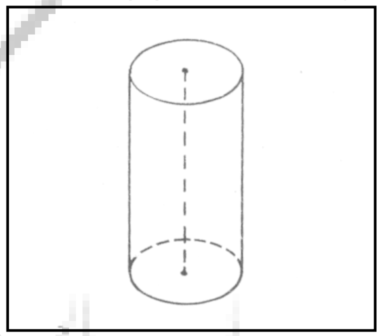
Area of side faces = (5 x 9 + 5 x 9 + 8 x 9) cm^2
= 45 + 45 + 72) cm^2
= 162 cm^2

Surface area = (24 + 162) cm^2
= 186 cm^2

Volume = Area of base x Height
= ½ x 8 x 3 x 9 cm^3
= 108 cm^3

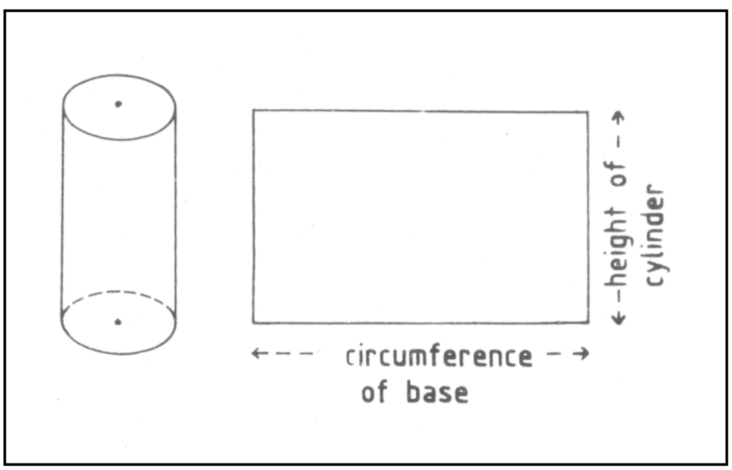
E. The Surface Area of a Right Cylinder

A cylinder is a prism with a circular base. A right cylinder has a congruent bottom and top, and the line through the midpoint of the top passes through the midpoint of the base.



Source: Mathematics Explained Std 8 – JJ de Wet

Consider the following diagram. The curve surface can be unfolded to form a rectangle. It is important to note that the rectangle has the same length as the circumference of the base and that the width of the rectangle is equal to the height of the cylinder.



Source: Mathematics Explained Std 8 – JJ de Wet

$\therefore$ The area of the curved surface
= The area of the rectangle
= L x B
= circumference of base x height of cylinder
= $2\pi r$ where r - represents the radius of the base and h the height of the cylinder.

The area of the base is equal to the area of the top.
Hence the area of the top and the base of the cylinder
= 2 x area of the base
= $2\pi r^2$ since the area of the base is equal to πr^2 where r represents the radius of the base.
$\therefore$ The surface area of the cylinder
= The area of the curved surface + The area of the bottom and the top
= $2\pi r$h + $2\pi r^2$
= $2\pi r$ (h + r)

Example:
Determine the surface area of a right cylinder on a circular base if the radius of the base of the cylinder is 14 cm and the height of the cylinder is 35 cm.
(Take π as $\frac{22}{7}$)
Surface area of the right cylinder
=$2\pi r$ (h + r)

=2 x $\frac{22}{7}$ x 14 (35 + 14) cm^2

= 2 x $\frac{22}{7}$ x 14 x 49 cm^2

= 2 x 22 x 2 x 49 cm^2

= 4 312 cm^2

Remember

1. The diameter of a circle is twice as long as the radius.
   D = 2r (Where d represents the diameter and r the radius of the circle)

2. The following formulae is used to calculate the circumference and area of a circle:
   C = $2\pi r$ (Where C represents the length of the circumference of the circle and r the radius of the circle)

   A = πr^2 (If A represents the area of the circle

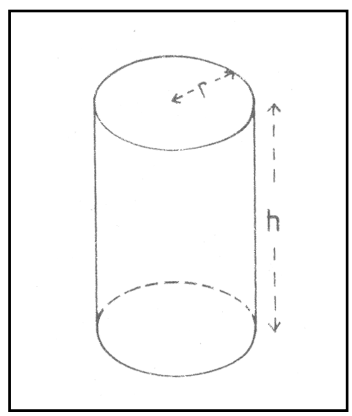
3. The surface area of a right cylinder on a circular base
   = $2\pi r$ (h + r) where r represents the radius of the circle and h the height of the circle.

4. The value of π is taken as $\frac{22}{7}$ unless otherwise mentioned.

F. The Volume of a Right Cylinder on a Circular Base

We have already shown that the volume of a right prism on a square or rectangular base is given by the product of the area of the base and the height of the prism.
With a more advanced knowledge of mathematics it can be proved that the volume of any right prism is always given by the product of the area of the base and the height of the prism. At this stage we accept the truth of this statement.

Source: Mathematics Explained Std 8 – JJ de Wet



The volume of the right cylinder
= area of the base x height of the cylinder
= πr^2h since the area of the base is given by πr^2 and the height by h.

Example:
Find the volume of a right cylinder with a height of 14cm if the diameter of the base is 14 cm.

2r = d
2r = 14 cm [d = 14cm]

Therefore r = 7cm

The volume of the cylinder

$= \pi r^2 h$

$= \frac{22}{7} \times 7^2 \times 14 \text{ cm}^3$

$= \frac{22}{7} \times 7 \times 7 \times 14 \text{ cm}^3$

$= 22 \times 7 \times 14 \text{ cm}^3$

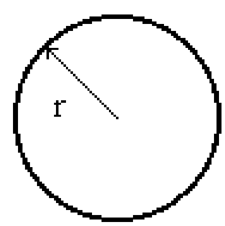
$= 2\ 156 \text{ cm}^3$

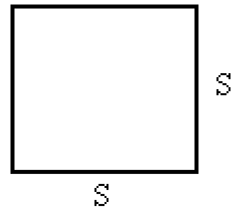
Remember

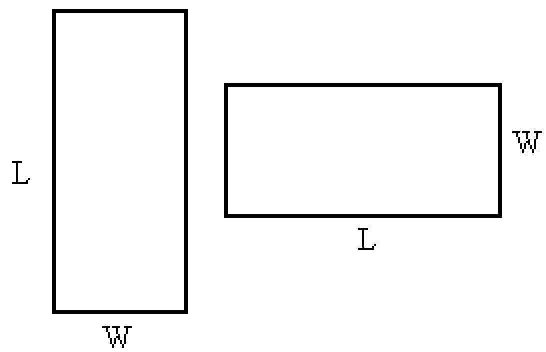
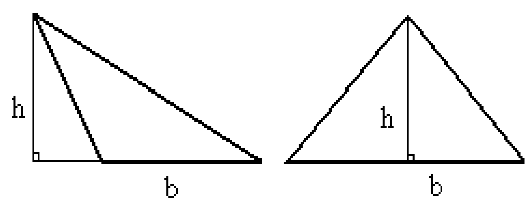
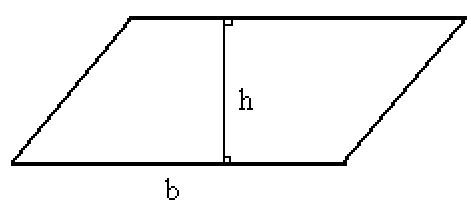
The volume of a right cylinder on a circular base = $\pi r^2 h$ where r represents the radius of the base and h the height of the cylinder.

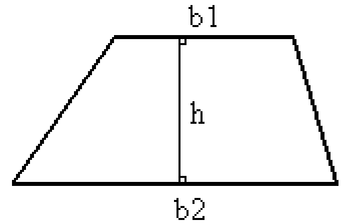
Calculating Volume and Surface Area

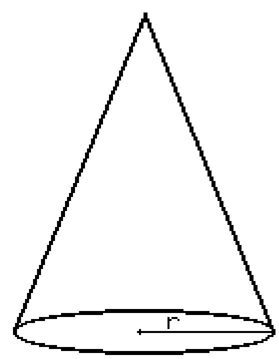
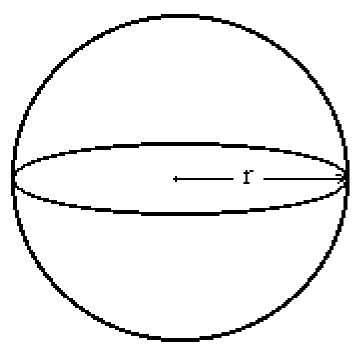
The table below gives the properties and formulae of all of the commonly occurring shapes.

1. **Circle** Area equals pi times the square of the radius: $A = \pi r^2$ Circumference equals pi times the diameter: $C = \pi d$ (Since the diameter is twice the radius, this formula can be written $C = 2\pi r$	
2. **Square** Area equals the square of the length of one side:	

$A = S^2$	

3. **Rectangle** Area equals length times width: $A = LW$ Perimeter equals twice the length plus twice the width: $P = 2L + 2W$	
4. **Triangle** Area equals one-half the base times the height: $A = \frac{1}{2}bh$	
5. **Parallelogram** Area equals base times height: $A = bh$	
6. **Trapezoid** Area equals one-half the height times	

the sum of the bases: $$A = \frac{1}{2}h(b_1 + b_2)$$	

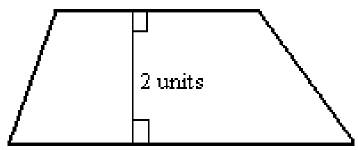
7. **<u>Cone</u>** The volume equals one-third of the base area times the height: $$V = \frac{1}{3}\pi r^2 h$$	
8. **<u>Sphere</u>** Surface area equals four times pi times the radius squared: $$A = 4\pi r^2$$ Volume equals four-thirds times pi time the radius cubed: $$V = \frac{4}{3}\pi r^3$$	

Sometimes it is necessary to calculate the surfaces areas and volumes of right prisms (i.e., end faces are polygons and the remaining faces are rectangles), cylinders and spheres.

The formulae for calculating the surface area and volume of a sphere are given in the table above. In order to calculate the volume of a right prism, the following formula is used:

Volume = (Area of polygon base) x (Height of Prism)

Example: Calculate the volume of a right prism that has a trapezoidal base as shown and a height of 10 units:



The area of the Trapezoid $= \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}\cdot 2\cdot(3+5)$

$= 8$ units^2

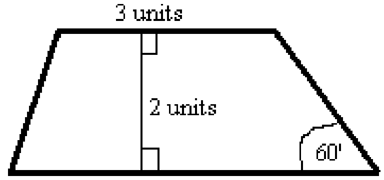
The volume is then the surface area multiplied by the height.

Volume = A.h
= 8 x 10
= 80 units^3

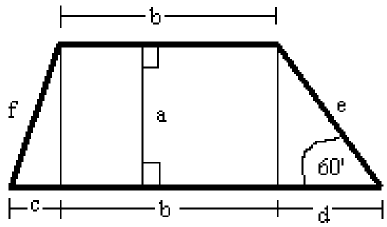
To calculate the surface area of a right prism, use the following steps:

1. Calculate the surface area of the base.
2. Calculate the perimeter of the base.
3. Multiply the perimeter of the base by the height of the prism and add twice the bottom surface area to it.

Example: Calculate the surface area of a right prism that has a trapezoidal base as shown and a height of 10 units:



This is quite complicated, as we need to use Pythagoras and trigonometry to calculate the perimeter of the Trapezoid. The perimeter is the sum of all four sides. We have the length of two of the sides already. Refer to the diagram to see how the other lengths are calculated.



$c + b + d = 5$ ....Given

$b = 3$ ....Given

$a = 2$ ....Given

now:

$$e = \frac{a}{\sin 60} = 2.31$$

$$d = e \cos 60 = 1.16$$

$$c = 5 - b - d = 0.84$$

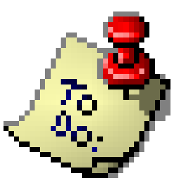
$$f = \sqrt{0.84^2 + 2^2} = 2.16$$

$$\text{Perimeter} = 2.b + e + d + c + f = 12.47$$

Area of sides = Perimeter x Height
= 12.47 x 10
= 22.47 units^2

Area of Base = Area of the Trapezoid
= 8 units^2 (…see previous example)

Surface area of right prism = (2 x 8) + 22.47
= **38.47 units^2**



Exercises

Individual Exercise 1

1. Using a ruler, find the following measurements:
2. The diameter of a R5 coin
3. The length of a pen or pencil
4. The width of your hand at the base of the fingers
5. The length of your eraser

Individual Exercise 2

The learner should explain in detail how to conduct measurements using the following measuring instruments:

- Micrometer
- Vernier calipers
- Measuring volume of a solid
- The lever arm balance
- A pipette

Explain what points are unique to each instrument and have to be borne in mind when taking measurements when taking measurements. Assessments will be based on the clarity of description of the process and the measurements

Individual Exercise 3

Get hold of a tin of dog food or the equivalent cylindrical object. Discuss how you would estimate the volumetric measurements of the tin by first doing a rough calculation in your head. Explain the steps you would take in your mental calculations and how you arrived at your answers. Give examples and then compare this against an actual measurement of the tin using a ruler. Compare your estimates with the actual. Assessments will be based on the clarity of description of the estimating and actual measurements

Individual Exercise 4

Learners will describe what instruments they would choose to carry out the following tasks, why they chose the instrument and describe in detail how they would:

1. Measure exactly 153 cc of water into a beaker
2. Weigh out 200 grams of salt onto a saucer
3. Measure the temperature of boiling water and a bucket of ice. The temperature at which water boils comes down by 2 degrees for every 1000 ft in altitude
4. Find the surface area of your desk at home
5. Find the area of one surface of a R5 coin

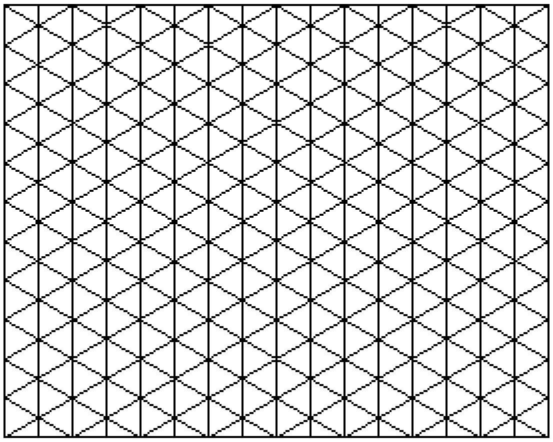
Assessments will be based on the clarity of description of the estimating and actual measurements

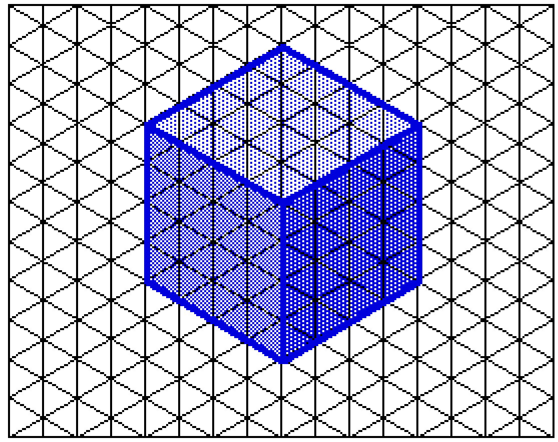
SPECIFIC OUTCOME 2

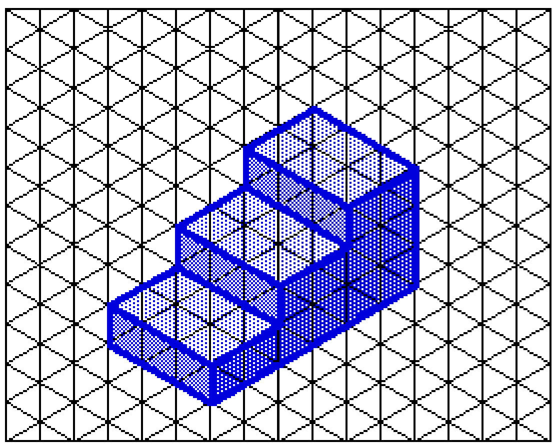
Explore, analyse & critique, describe & represent, interpret & justify geometrical relationships.

Technical Drawing
Isometric drawings

Isometric drawing is one of the possible views that can be used to represent 3-D drawings. An isometric drawing is drawn on three different types of lines: vertical lines, 30° lines to the left and 30° lines to the right. An easy way to draw 3-dimentional objects in isometric is to use isometric paper. Isometric paper is very similar to graph paper, but instead of horizontal lines and vertical lines, it has vertical lines, 30° lines to the left and 30° lines to the right. A sample piece of isometric paper is given below. On the right of the figure is an example of how a 3 dimensional cube would be drawn on the paper.







3 stairs

Isometric Paper with Examples of Objects

Isometric paper can be used to draw rough sketches to represent objects and situations. In order to accurately represent objects, one would need to use proper drawing equipment or computer packages.

To draw isometric objects accurately, it is best to use a technical drawing board with drawing instruments such as setsquares, compasses and rulers. Technical drawings that would be messy on isometric paper can be accomplished with the drawing instruments mentioned. The most common type of drawing board is generally capable of handling paper sizes up to and including A3. Draftsmen in the professional environments may use technical drawing boards up to size A0.

Draftsmen will also often use technical drawing software instead of the more traditional drawing instruments. There are many programs available, such as

AutoCAD, CADDIE, Solid Edge, etc. These are all very powerful tools, however, some are quite complicated to use and students wishing to learn these programs may have to attend special courses specific to each program.

Scale

All technical drawings that have dimensions should have a scale. The scale tells us how the size of the drawing on the paper is related to the size of the object in real life. Scales are represented as a ratio of two numbers. For example, a scale of 1:10 on a drawing means that 1 unit on the drawing is equal to 10 units in real life. Or in other words, the drawing is 10 times smaller that the real thing. The inverse applies for small real life objects being represented larger than they are. For example a 100:1 scale means that the distances in the drawing are 100 times larger than on the object.

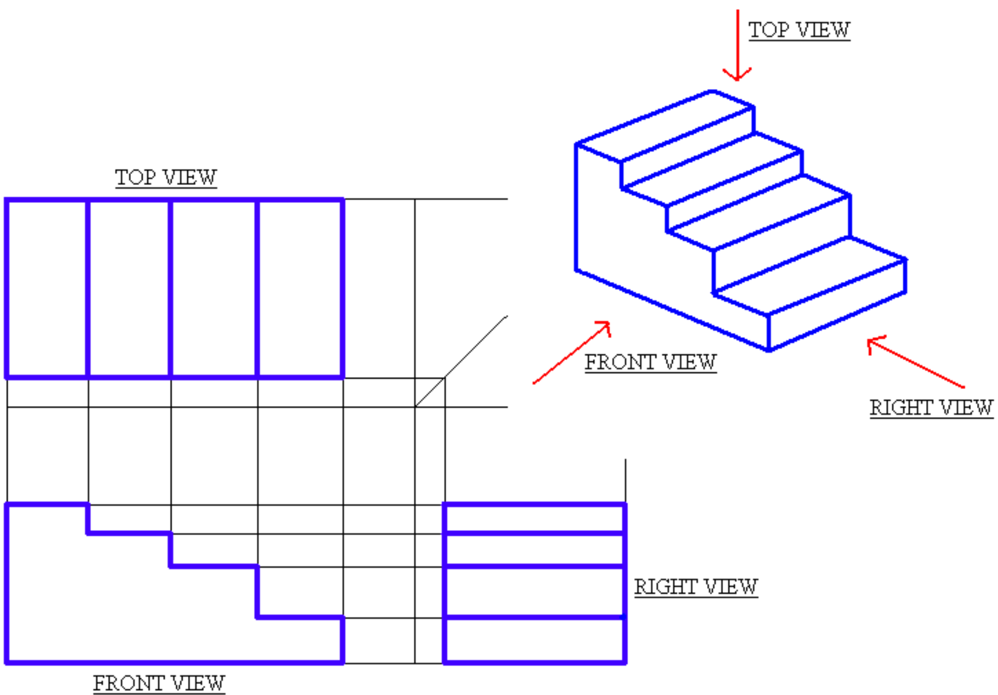
Example: You are required to machine a part on a metal lathe. The drawing of the part is at a scale of 10:1. You want to machine a round section on a shaft. On the drawing the diameter of the shaft is 100-mm. How large should the shaft really be.

Size of real object = 100/10
= 10 mm

The shaft should be machined to a diameter of 10 mm.

Orthographic drawings
Orthographic views are two-dimensional views of three-dimensional objects. Orthographic views represent the exact shape of an object as seen from one side at a time as you are looking perpendicularly to it. Depth is not represented in an orthographic drawing.

An orthographic drawing contains three 2-dimensional views of a 3-dimensional object. An orthographic object will contain a front top and side view. There should be enough information in the drawing such that the 3-dimensional object could be drawn from what is given. The figure below illustrates an orthographic drawing of a 3-dimensional object. Note that the three views are labelled. As with isometric drawings, hidden detail is drawn as a dashed line and circles centres are drawn in a long-short-long dashed line.



Orthographic drawing of an Isometric Object

The three views can easily be assembled to represent the object in three dimensions. By looking at the figure above, it is possible to see how this process is accomplished. It is important to try and visualise the object before you start drawing, this may help eliminate gross errors.

Maps and Time Zones

Land maps often need to be interpreted in order to plan or negotiate routes and developments. When interpreting a map the three most important items to remember are:

- Direction
  Maps should have a symbol indicating which direction is north. This is used to calculate bearings and directions.

- Co-ordinate system
  Along two edges of the map there should be co-ordinate to describe positions in the map relative to the rest of the world. These co-ordinates will be of the form degrees, minutes, seconds and represented (…° …' …"). Two co-ordinates can represent any point on the map. One to show how far north or south (…° …' …" N/S) you are from the equator and the other shows how far east or west you are from the Greenwich Meridian (…° …' …" E/W).

- Scale
  This allows one to convert distances on the map to distances on the ground.

- Key
  Every map should have a key to help interpret the symbols that occur on the map. For example a different type or colour line will represent roads, boundaries, and rivers.

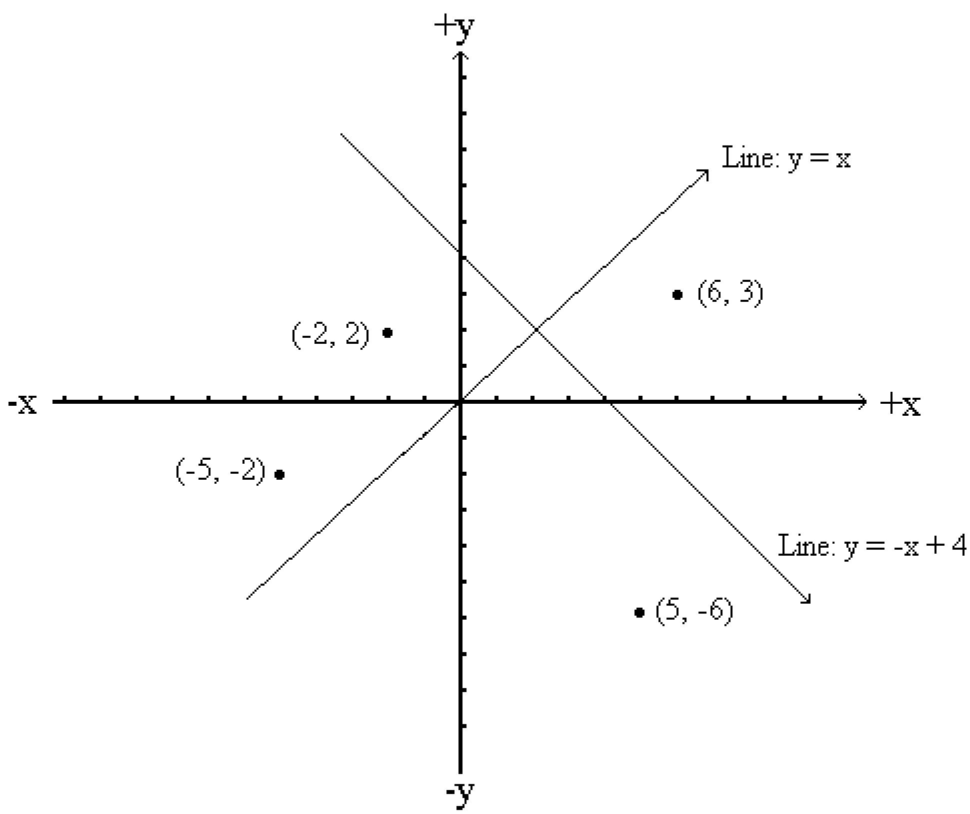
- Projection
  Trying to project a curved surface (the world) onto a flat surface (a map) results in certain distortions. Different types of projections have been created that conserve different properties of the map. For example, certain projections may conserve area and distance while others conserve distance and direction.

If all of the above components are present, interpreting maps can be very straightforward. Simply make accurate measurements and consult the key for identification of symbols.

Time zones have been created because of different parts of the world experiencing night and day at the same "time". All time is based around the Greenwich meridian (GM), which runs north to south approximately through London. Any point to the east of the GM is "ahead" in time and any point to the west is "behind" in time. For every 30 degrees one travels east, you move two hours ahead in time. For every 30 degrees west, you move backwards in time.

For example, you caught a flight from Cape Town (GMT +2 hours) to New York (GMT –5 hours). If the flight left at midday, and it took 10 hours to fly to New York. In South Africa by the time you landed in New York, your body would think its 10 0-clock at night. But in reality, when you land in New York the time would only be 3 o-clock in the afternoon! That is how people suffer from jet lag.

Cartesian Co-ordinates:
A Cartesian Co-ordinate system is one in which points on a plane are identified by an ordered pair of numbers, representing the distances to two or three perpendicular axes. The number of axes determines the dimension, two axes for 2-dimentional space and three axes for 3-dimensional space. The diagram below shows a 2 dimensional Cartesian co-ordinate system. The horizontal axis is called the ***x-axis*** and the vertical axis is called the ***y-axis***.



The Cartesian co-ordinate system can be used to describe points and lines. The lines can be straight or curved and can be used to represent direction or perhaps the locus (path of a point). The point where the ***x-*** and ***y-axes*** meet is called the origin. In terms of ***x***, anything to the right of the ***y-axis*** is positive, and anything to the left is negative. In terms of ***y***, anything above the ***x-axis*** is positive, and anything below is negative.

Representing points: Any point is represented in brackets with the x-coordinate first, and the y-coordinate second

Eg: (x, y). There are some examples of points in the figure above.

Any world map is an example of a Cartesian co-ordinate system. The origin would be the intersection of the equator ad the Greenwich meridian, and instead of *x* and *y*, it is North, South, East and West.

An important point to remember when determining solution to mathematical and technical problems is to always answer the problem in the same systematic way. This helps eliminate simple errors and helps establish a method to solving problems. Always write down the question in a summarised form. Then when rearranging equations and numbers, make sure that every step is written down. Don't try to calculate three steps in your head and then write down the answer to the fourth step. Errors are likely to occur if this is done. Remember as well that working neatly will always make it easier to find and correct mistakes.

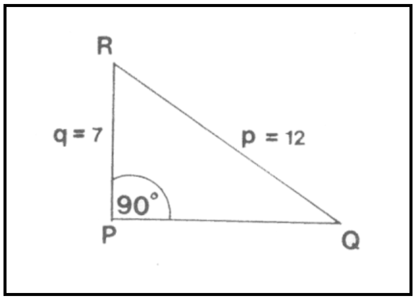
When determining distances, either by an estimate or a measurement, remember the accuracy required in the problem. It is inappropriate to measure or estimate distances to the nearest millimetre if the problem does not require such accuracy. Conjectures that are made in the analysis part of a problem need to be well planned before they can be regarded as acceptable. When doing calculations that are based on an estimate, the estimates need to be accurate otherwise the solution may be greatly incorrect. It is good practice to state the accuracy or confidence with which estimates are made. The estimates also need to be based on geometrical principals. Such properties can be found in the table that describes and represents the properties of shapes.

In the problem solving process, it is often advisable to try and generate more than one strategy that could be used to solve the problem. These different strategies can then be compared against one another in terms of appropriateness, complexity and viability. Once the best strategy has been selected and the problem solved one can use hindsight to identify the strengths and weaknesses of the chosen strategy as well of those that were not chosen. This process will improve the problem solving technique and hopefully assist you in not making the wrong assumptions in similar future problems.

Exercises

Individual Exercise 1

Questions related to the Pythagoras Theorem



In $\Delta PQR,\ \angle Q = 90°$

Calculate the length of the third side in each of the following cases. Leave the answer in square root form when the square root cannot be easily calculated.

a)	PR = 15m	and	QR = 9m
b)	PR = 20m	and	PQ = 16m
c)	PQ = 13cm	and	QR = 12cm
d)	PR = 26cm	and	QR = 24cm
e)	PR = 50mm	and	PQ = 22mm

Individual Exercise 2

An elderly man travelled by ox-wagon from the village of Estcourt to hamlet of Mooi River. The journey took 2 days. His odometer, which was very primitive, had an accuracy of ± 20%. If his odometer told him that the journey was 35 kilometres and if he took twelve hours to get there, what were his three possible velocities, in meters per second? (Hint: calculate his fastest, slowest and average speeds)

Individual Exercise 3

Calculate the surface area of a right prism with a square base having:

a) A length of 8cm and a height of 12cm
b) A length of 15m and a height of 21m
c) A length of 7m and a height of 11m

Calculate the cost to manufacture a tank in the form of a right prism with a square base having a length 4m and a height of 12m if the material used cost R58 per square metre.

Calculate the surface area of the following right prisms:

L=6m	B=5m	H=8m
L=2,9m	B=1,3m	H=2,1m
L=9,5m	B=8,3m	H=12,8m

Individual Exercise 4

Calculate the volumes of rectangular prisms with the following dimensions:

Length	Width	Height
a) 4m	2m	1,5m
b) 38mm	8cm	4,2cm
c) 8,2m	6,3m	6m

A box is 20mm long and 15mm wide

a) What is the area of the base?
b) Find the volume of the box if it is 25mm high

The volume of a rectangular box is 640cm^3. Calculate its height if the area of the base is 160cm^2.

The volume of a rectangular box is 500cm^3. Calculate the area of the base if the height of the box is 20cm.

How many rectangular bricks with dimensions 240mm, 125mm and 80mm are necessary to build a wall which is 20m long, 2m high and 240mm thick?

Individual Exercise 5

Find the surface areas of the following right cylinders on circular bases:

	Radius of base	Height
a.	7cm	14cm
b.	28mm	35mm
c.	4m	3m

Find the surface area of a right cylinder on a circular base if the diameter of the base of the cylinder is 14cm and the height of the cylinder is 35cm.

Calculate the volume of the following right cylinders on circular bases:

Radius of base	Height of cylinder
2m	7m

1,5m 5m
54mm 35mm

Calculate the volume of a right cylinder on a circular base if the diameter of the base is 21cm and the height of the cylinder is 10cm.

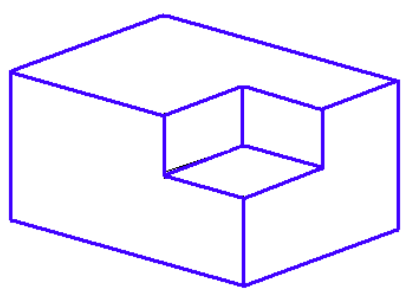
Individual Exercise 6

Choose a 15 x 15 meter area that is convenient for all the members of the group to meet at. Draw a detailed map of the area by using a 20-meter tape. The map should include symbols for spatial features, a scale, a direction arrow and a key identifying the features. Also form a Cartesian co-ordinate system for your map whereby all points on the map are referenced to some sort of origin (Eg, a doorway, tree, concrete pillar, etc.)

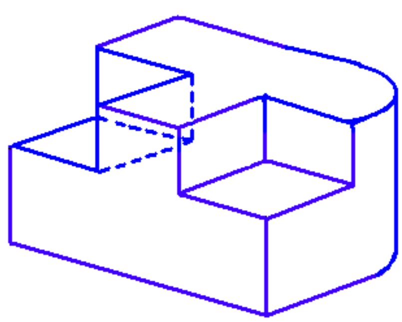
Individual Exercise 7

Represent the following three-dimensional objects as three orthographic views, namely top front and side view. The views need not be accurate; it's an exercise to ensuring that the principals behind the process are understood.

a.)



b.)



Formative Assessment Activity 2

Please open your Portfolio of evidence (Logbook) and complete the activities for this specific outcome, following the instructions found in your logbook.

Your facilitator will advise you as to the requirements and deadlines for completing and handing in these formative assessments. Write these instructions here:

Unit Standard 7468
Use mathematics to investigate and monitor the financial aspects of personal, business and national issues

When you have completed this section, you will be able to understand and explain the following:

- Use mathematics to plan and control financial instruments including insurance and assurance, unit trusts, stock exchange dealings, options, futures and bonds
- Use simple and compound interest to make sense of and define a variety of situations including mortgage loans, hire purchase, present values, annuities and sinking funds
- Investigate various aspects of costs and revenue including marginal costs, marginal revenue and optimisation of profit
- Use mathematics to debate aspects of the national and global economy, including tax, productivity and the equitable distribution of resources.

This section of learning is designed to assist the learner to gain the basic life skills required to do and/or understand a variety of calculations that might be required as an adult. This will help with insurance, investments, savings and business ventures.

In completion you should be able to use mathematics to plan and control financial instruments, use simple and compound interest to make sense of and define a variety of life situations, investigate various aspects of costs and revenue, and finally use mathematics to debate aspects of the national and global economy.

Specific Outcome 1:
Use mathematics to plan and control financial instruments such as insurance, assurance, unit trusts, stock exchange dealings, options, futures and bonds.

Insurance and assurance

Insurance is a product that many people in South Africa have no exposure to. They are completely oblivious to the costs and the benefits of this financial product. Essentially, insurance is purchased by private individuals as well as by businesses. An individual may insure his car, motorcycle, as well as his home and the contents thereof. A business would similarly insure all its assets. The requirement to insure these items is availability of disposable income and the acceptance of the home or business owner of the insurance company's offer.

Insurance companies normally charge a percentage of the insured value of the items to be insured. So assuming the value of the item is R 1000, and the insurance company wants 7% as an annual fee, the premium on this item would be R 70 per annum. The owner of the item would then have to decide whether cost of replacing the item and the risk of the item being stolen is worth the cost of the premium. If the owner wishes, he can request the insurance broker to supply three quotations from three different insurance companies in order to ensure that the deal offered is reasonable.

The price will vary according to the risk. The insurance company will have a good understanding of the risk in relation to the item insured. Young men and cars/motorbikes are high risk. Families living in high risk suburbs with high burglary rates are high risk. Old men riding Harley Davidson motorbikes or Ferrari sports cars are low risk.

Life insurance is commonly known as assurance. This is when someone's life is insured. This means that in the event of the insured person's death, the beneficiary will receive the sum assured. The system works in exactly the same way as for asset insurance. The only difference is that the risk is now based upon the age of the person assured, or alternatively the assured medical condition, smoking habits and any other medical issues which could reduce the life span of the insured, as well as sporting activities such as parachuting or mountain climbing.

Unit trusts

Unit trusts are a simple way for the man-in-the-street to invest in shares on the stock exchange. Normally an individual would contact a stock broker, and once agreeing on an amount to be invested, the stock broker would then invest on behalf of the individual, or alternatively would advise the individual on what to invest in and then once agreement had been reached the stock broker would invest on the individual's behalf. The only problem is that most stock brokers have now set a minimum investment level, and they will not invest on behalf of individuals who have less than this amount to invest.

This leaves the individual who wants to invest a small amount of money, with no options except the purchase of unit trusts. An organisation in the financial services sector will put together a share portfolio, perhaps within a particular sector. They will then sell this shareholding off in smaller portions. They also purchase back these smaller portions from people who do not want to retain them any longer. This makes it simple and low cost for those people who want to invest in shares but who do not have the minimum stock broking investment amount. All share purchases and sales are done through a stock exchange, which in the South African case would be the Johannesburg Stock Exchange, commonly known as the JSE.

Options and futures

Options and futures and normally traded on the Futures exchange. These are bets that are really intelligent estimates on commodity prices. Individuals and organisations will estimate what the price of a commodity, say wheat, will be on a certain date.

Options

A privilege sold by one party to another that offers the buyer the right, but not the obligation, to buy (call) or sell (put) a security at an agreed-upon price during a certain period of time or on a specific date. Options are extremely versatile securities that can be used in many different ways. Traders use options to speculate, which is a relatively risky practice, while hedgers use options to reduce the risk of holding an asset.

Futures

A financial contract obligating the buyer to purchase an asset, or the seller to sell an asset, such as a physical commodity or a financial instrument, at a predetermined future date and price. Futures contracts detail the quality and quantity of the underlying asset; they are standardized to facilitate trading on a futures exchange. Some futures contracts may call for physical delivery of the asset, while others are settled in cash. The futures markets are characterized by the ability to use very high leverage relative to stock markets.
Futures can be used either to hedge or to speculate on the price movement of the underlying asset. For example, a producer of corn could use futures to lock in a certain price and reduce risk (hedge). On the other hand, anybody could speculate on the price movement of corn by going long or short using futures.

The primary difference between options and futures is that options give the holder the *right* to buy or sell the underlying asset at expiration, while the holder of a futures contract is *obligated* to fulfill the terms of his/her contract.

In real life, the actual delivery rate of the underlying goods specified in futures contracts is very low. This is a result of the fact that the hedging or speculating benefits of the contracts can be had largely without actually holding the contract until expiry and delivering the good(s). For example, if you were long in a futures contract, you could go short the same type of contract to offset your position. This serves to exit your position, much like selling a stock in the equity markets would close a trade.

Call
The period of time between the opening and closing of some future markets, wherein the prices are established through an auction process. An option contract giving the owner the right (but not the obligation) to buy a specified amount of an underlying security at a specified price within a specified time. In some exchanges the call period is an important time in which to match and execute a large number of orders before opening and closing. A call becomes more valuable as the price of the underlying asset (stock) appreciates.

Put
An option contract giving the owner the right, but not the obligation, to sell a specified amount of an underlying security at a specified price within a specified time. The act of exercising a put option. A put becomes more valuable as the price of the underlying stock depreciates.
Options and futures are complex financial instruments and not for the faint-hearted or uninformed. These are ideally suited to individuals or organisations who have been widely exposed in this area.

Bonds
The government offers bonds which investors can purchase. These bonds normally offer a higher interest rate than the bank, also have a minimum investment amount, and are guaranteed by the government. These are often traded internationally, especially when interest rates are very low in a developed country, and poor returns are available to their investors. They may then seek bonds in secure developing countries. A bond will normally have a life span with a guaranteed rate of return (interest rate).

Public companies (those listed on the stock exchange) may also issue bonds. These bonds are very dependent on how well run the business is. This is essentially no different to the way a government runs a country. A business may issue bonds which are initially well rated, but over time the poor management of the business could result in the business' bonds being re-rated as junk bonds, i.e. worthless bonds.

Specific Outcome 2:
Use simple and compound interest to make sense of and define a variety of situations such as mortgage loans, hire purchase, present values, annuities and

sinking funds.

Simple Interest
When money is borrowed, interest is charged for the use of that money for a certain period of time. When the money is paid back, the principal (amount of money that was borrowed) and the interest is paid back. The amount of interest depends on the interest rate, the amount of money borrowed (principal), the length of time that the money is borrowed and the method of calculating the interest.

The formula for finding simple interest is:

Interest = Principal * Rate * Time

If R100 was borrowed for 2 years at a 10% per annum (per year) interest rate, the interest would be R100 * 10 / 100 * 2 = R20. The total amount that would be due would be R100 + R20 = R120.

Simple interest is generally charged for borrowing money for short periods of time. Compound interest is similar but the total amount due at the end of each period is calculated and further interest is charged against both the original principal and also the interest that was earned during that period. Normally this form of interest is only charged by friends. Most businesses will charge compound interest.

Compound Interest
When you borrow money from a bank, you pay interest. Interest is really a fee charged for lending the money, or the lender's profit for doing so; it is a percentage charged on the principle amount for a period of time, in most cases the period would be a year or a multiple thereof.

If you want to know how much interest you will earn on your investment, or if you want to know how much you will pay above the original amount of the loan, you will need to understand how compound interest works.

Compound interest is paid on the original principal **and** on the accumulated past interest.

Formula:
P is the principal (the initial amount you borrow or deposit) ;
r is the annual rate of interest (percentage as a decimal eg. 18% =0.18);
n is the number of years the amount is deposited or borrowed for;
A is the amount of money accumulated after n years, including interest.

When the interest is compounded once a year:

$$\mathbf{A = P(1 + r)^n}$$

However, if you borrow for 5 years the formula will look like:

$$\mathbf{A = P(1 + r)}^5$$

This formula applies to both money invested and money borrowed.

Frequent Compounding of Interest:
What if interest is paid / charged more frequently?

Here are a few examples of the formula:

Annually	$= \mathbf{P} \times (1 + r)$	= (annual compounding)
Quarterly	$= \mathbf{P} \times (1 + r/4)^4$	= (quarterly compounding)
Monthly	$= \mathbf{P} \times (1 + r/12)^{12}$	= (monthly compounding)

Difference between simple and compound interest
The following two examples of methods of computing interest on a principal sum illustrate the differences between simple interest and compound interest. Both examples use a R100 principal and 7% interest.

Simple Interest
The interest rate is applied only to the original principal amount in computing the amount of interest.

Year	Principal(P)	Interest(R)	Ending Balance(A)
1	100.00	7.00	107.00
2	100.00	7.00	114.00
3	100.00	7.00	121.00
4	100.00	7.00	128.00
5	100.00	7.00	135.00

Total interest 35.00

Compound Interest

The interest rate is applied to the original principle and any accumulated interest.

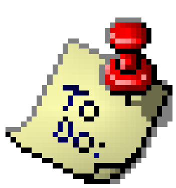
Year	Principal(P)	Interest(R)	Ending Balance(A)
1	100.00	7.00	107.00
2	107.00	7.49	114.49
3	114.49	8.01	122.50
4	122.50	8.58	131.08
5	131.08	9.18	140.26
	Total interest 40.26		

All other things being equal, compound interest has a larger effect as the time period increases and as the interest rate increases.

The effect of time is an important concept to remember. This is important when investing and when buying property. If you have purchased property with a bank bond (mortgage bond), generally the period of repayment is between 20 and 30 years. However, if you increase your monthly repayment, you can dramatically reduce the interest payable. As an exercise, go to a bank and request a R500 000 bond on a property over 25 years, and enquire what the payment details would be for this bond. Then ask them how long it would take if you increased your payment by R 1000.00 per month. You will be surprised by the amount of time the repayment period is reduced. Similarly take the first amount and multiply x 25 years and then multiply by 12 months to calculate the total paid. Then do the same with the increased repayment over the shorter period. You will be stunned at how much money you will save.

However, the same holds true in reverse. The longer you invest for the more your money grows with interest. That is why the sooner/younger you begin to invest, the sooner you have a very substantial investment.

The effect of compound interest is phenomenal whether you are borrowing or investing. So when investing - start soon, and keep investing as much as possible, but, when borrowing - pay back as soon as possible.



Exercise 1

With your partner, complete the following exercise:

a. Calculate the amount of the repayments on a new car costing R100000.00 purchased on hire purchase over 60 months at 15% per month. Calculate interest due after payments received on a monthly basis. Advice the interest amount paid and reflect the total interest amount as a percentage of the original amount borrowed.

b. Calculate the repayments on a R500000.00 home bond (mortgage bond) over 20 years at 12%. Calculate interest due after payments received on a monthly basis. Calculate total interest paid over the 20 years. See the effect on interest paid if you increase the repayment by additional 20% per month. Calculate the total interest after increasing the repayment. Compare the two figures as well as the impact of the increased repayment amount on the time required to repay the home bond.

Specific Outcome 3:
Investigate various aspects of costs and revenue, incorporating marginal costs, marginal revenue and optimisation of profit

Cost and Revenue
Traditionally cost in a retail environment has been calculated on the basis of what the item cost to purchase, then a markup was added and the item resold at the higher price. Similarly if the product being sold was a service, then the time taken to implement or deliver the service is measured and a pro rate cost at the going labour rate is calculated in order to establish the cost of the service.

A combination of these two systems is sometimes used in manufacturing systems, whereby the cost of raw material is tracked and measured on a pro rate consumption basis, together with a pro rate labour costs, and also a pro rate machine cost. These three sources of cost were added together and a cost was calculated for the item manufactured. This is called absorption costing.

An alternative to this is to use marginal costing in the manufacturing sector.

Marginal Cost
Marginal cost is defined as

$$MC=\frac{\Delta C}{\Delta Q}$$

As usual, Q stands for (quantity of) output and C for cost, so ΔQ stands for the change in output quantity, while ΔC stands for the change in cost per unit. As usual, marginal cost can be interpreted as the additional cost of producing just one more ("marginal") unit of output.

Let's have a numerical example of the Marginal Cost definition to help make it clear. In the John Bates Clark style example we have been using, total cost is R280 000 for an output of 3 120 units, and it is R330 000 for an output of 3625. So we have:

$$\Delta C = 330000-280000=50000$$

and

$$\Delta Q = 3625-3120=505$$

so that

$$\frac{\Delta C}{\Delta Q} = \frac{50000}{505} = 99.01$$

a marginal cost of R 99.01 for the next unit produced. As usual, this is an approximation, and the smaller the change in output we use, the better the approximation is.

Marginal revenue

The marginal revenue is the difference between the selling price of a single unit less the variable costs of a single unit.

The revenue associated with one additional unit of production. Whether this is higher, lower or the same as the revenue from the previous unit of production depends on the demand for the producer's product. In the case of a producer who supplies a very small percentage of the market, an extra unit of production is unlikely to have an effect on market prices. In this case, increased production will not affect marginal revenue. On the other hand, if the producer supplies most or all of the market, then increased production is likely to reduce marginal revenue.

When calculating marginal cost or revenue it is important to use correct values and calculation methods.

The key difference between absorption and marginal costing is that with absorption costing, all fixed and variable costs are calculated as part of the unit cost. This means that for every unit manufactured the total cost is slightly less. Because of the absorption of fixed manufacturing overheads into the value of work-in-progress and finished goods stocks. If stocks remain at the end of an accounting period, then the fixed manufacturing overhead costs included within the stock valuation will be transferred to the following period.

However with marginal costing the fixed costs are established and the marginal revenue per unit is similarly established. Using these two figures they can establish the breakeven quantity to be manufactured. Then they set the desired profit required in Rand and once again use the marginal revenue to calculate the additional quantity to be manufactured in order to achieve the desired profitability.

This does get a little complex and requires linear programming skills once a wider range of products is manufactured.

Specific Outcome 4:
Use mathematics to debate aspects of the national and global economy including exchange rates, imports, exports, and comparative effectiveness of currency in relation to remuneration, monetary policy and the control of inflation.

Countries function just like very big businesses. Each country has a national treasury, which is the department which control the allocation and spending of all government departments. They also decide on the country's financial (fiscal) policies. National budgets are published by the government on an annual basis and they are normally released in February of each year. Included in the budget are the incomes and expenditures that the country expects to receive and spend in the coming year and the budgets (allowances) that each department is given. These budgets are divided up by the Treasury on the basis of where the deemed greatest needs are.

National income is the money that is received primarily from taxation of businesses and individuals. Based upon expected incomes and required expenditure, the Treasury will decide whether to retain existing tax rates, or increase or decrease them. There are different tax brackets that individuals fall into, for example a person who is earning between R0 and R74 000 might be taxed at 18%, the second tax bracket for people earning between R74 001 and R115 000 the employee might be taxed at 25%. The maximum percentage tax that employees might be taxed at is 40%. Tax is also due to be paid on importing and exporting goods, and tax is also added on when purchasing items. This particular form of tax is referred to as VAT, this stands for Value Added Tax which is currently standing at 14%.

The national expenditure that the governments are responsible for includes education, health services, roads, road signs, police services, the national defense force, provision of water and electricity to city councils. Each of these are budgeted for in the national budget and the expenditure on these is limited.

Other key elements that are used in regional and national budgets are:

- Gross domestic product (national income);
- Gross geographic product (regional income);
- Balance of payments.

The gross domestic product (GDP) in any country refers to the total value of all the final goods and services that are produced in a country in a given year. Included in the budget is the GDP from the previous year as well as the predictions for the following year.

A budgetary surplus occurs when income exceeds expenditure. This can occur if there is a conscious effort by the Treasury to spend less than they are expecting as income, or when, as occurred in South Africa in the early 2000's the government collected a lot more taxes than was expected. A deficit occurs when a government spends more than they budgeted to, or collects less than they expected to. In a period when a government needs to do major infrastructural work, such as build a lot of new roads, dams, schools, power stations, etc, and does not have the cash to do so, they might deliberately go into deficit and would borrow from international banks. The current rule of thumb is that the deficit should not exceed more than 4% of income.

Controlling the money supply in the economy is a task that is performed by the central bank, known in South Africa as the South African Reserve Bank (SARB). They have the ability to influence economic activity through the monetary policy. The SARB can control the supply of money that is in the economy by either reducing or increasing the amount of reserves in the banking system. When the money base increases in the economy, more loans are made possible by commercial banks and thus more expenditure is done by the people who are granted the loans. This stimulates economic growth.

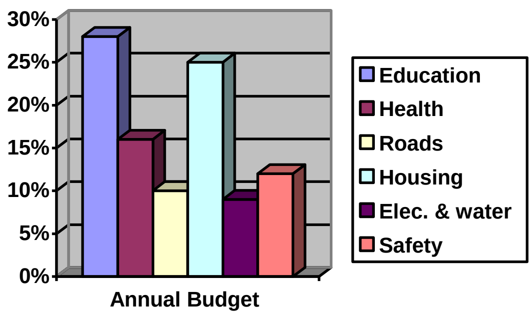
There are many mathematical calculations that have to be carried out in the economy. The computational tools that are used for these calculations are models that have been used to represent the past performance and assist in predicting future performance. Actual tools that are used are spreadsheets, graphs, tables and economic theories that are used to plot the expected budgets according to past incomes and expenditures. Past trends are an efficient tool that can be used to anticipate future incomes and expenditures.

A budget is an indication of the expected income and expenditure for the given period of time. Budgets can be represented by using a number of different methods. Most of these methods include an illustration of some sort, for example a pie chart, graphs, tables and formulae. Some of these illustrations will be shown in the following paragraph.

Education and training	28 %
Health services	16 %
Roads and infrastructure	10 %
Housing	25 %
Electricity and water	9 %
Safety and security	12 %

The diagram above is referred to as a pie chart or a pie graph. The budget can be illustrated using a diagram of this nature by dividing the pie into different portions that represent the amount that the government can spend on each of the services. Included in a pie chart are normally the percentages of the budget that are allocated to each of the various expenditure items. These percentages are reflected on the left hand side of the pie chart.

The same figures can be illustrated on a bar graph. This is shown below.



This bar graph represents the different items that the annual national and regional budgets are spent on. It is easy quick and easy to read and gives an indication of the expenditure on the various items relative to other expenditure items.

A table of the national budget can also be illustrated below.

Expenditure item	**Percentage of the budget**
Education and training	28 %
Health services	16 %
Roads and infrastructure	10 %
Housing	25 %
Electricity and water	9 %
Safety and security	12%

When the actual income and expenditure for the given year has been recorded, it is possible to compare this to the budget. Often the budget and the actual income and expenditure do not correlate, thus variances can be identified. The difference between the actual income and expenditure and the budget can be calculated by using subtraction. For example to determine the variance in the spending on health services, the actual spending on health services should be subtracted from the budget. If the answer is a positive figure, the amount that was spent was less than the amount that was budgeted for and vice versa.

Inflationary effects

Inflation

The term inflation refers to a sustained increase in the prices levels of goods and services. There are two main types of inflation:

1. Demand-pull inflation
2. Cost-push inflation

Demand-pull inflation is caused by an excessive demand for goods and services, which pushes the prices upwards. Cost-push inflation is when costs increase, for example energy price increase is imposed by a wage increase demanded by labour unions or the price of oil could push the price of petrol up which in turn pushes up the price of transport which in turn pushes the price of all products up.

The increase in prices is normally measured by using the consumer price index (CPI). This is used to show the increase in the prices of consumer goods and services. When the price of consumer goods and services rises by 10 %, the CPI will also rise by 10 %. The method of calculation used to calculate the CPI is by observing the changes in the cost of purchasing a standardised bundle of consumer goods and services. When the cost of buying this standardised bundle of goods and services rises, the consumer price index also rises. When the cost falls the CPI falls too.

CPI is calculated using this formula:

$$\text{CPI} = \frac{\text{Current cost of typical bundle}}{\text{Base year cost of typical bundle}} \times 100$$

The base year cost of a typical bundle is calculated using the figures say from the 1982 period. The CPI is usually clearly defined as to what the base year is.

The formula for inflation is:

Inflation = Percentage change in CPI

The actual increase in the general price level is referred to as the inflation rate and this is measured in percentages. When inflation is targeted, measures are taken to reduce the current levels of inflation. Inflation targeting refers to government's efforts and commitment to reducing the rate of inflation and this can be done by doing one of two things. These are:

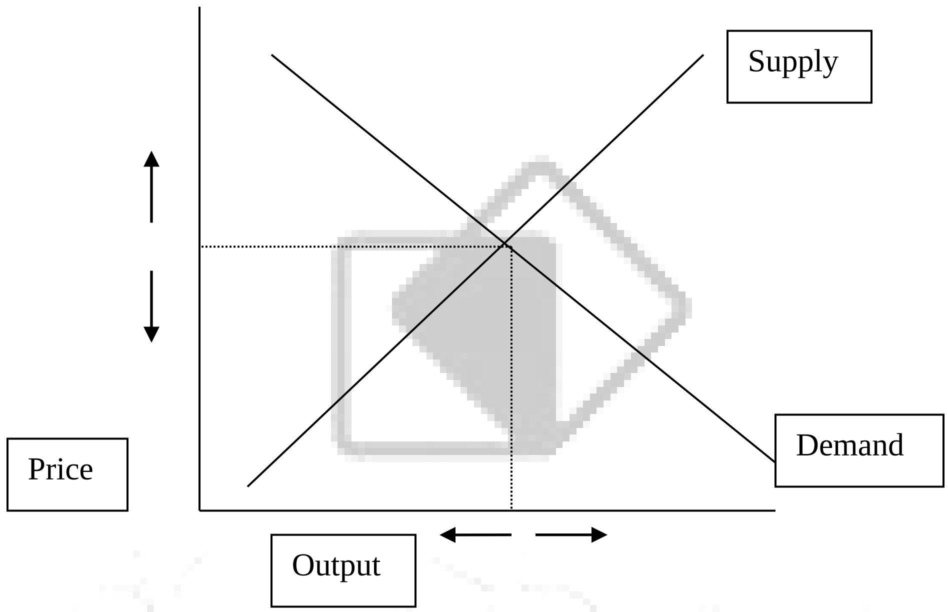
- Demand side

The total amount of spending in the economy must be reduced. By doing this the aggregate demand for goods and services decreases and the prices charged for these goods and services can be reduced. The demand curve shifts to the left and prices fall. This is done by controlling the amount of available money in the financial system by raising or lowering the interest rate of money lent to commercial banks by the central bank, known as the repo rate.

- Supply side

Policies should be developed that lead organisations to increase their capital equipment which results in additional production of goods and services. The supply curve shifts to the right and the price of goods and services falls.

The diagram below can be used to understand this better.



When individuals and businesses deposit their money into the bank they are due to receive interest on this money. The interest is normally calculated on a monthly basis and is added to the investors bank balance. Individuals and organisations can borrow money from the bank, but they are required to pay interest on the money that they borrow. The interest that is charged on borrowing money varies, but it is normally around a number of percentage points above the report rate. This interest is normally charged on a monthly basis and is reflected in the client’s statement.

The repo rate refers to the interest rate that commercial banks are charged for borrowing sums of money from the Reserve Bank. The Reserve Bank is the central bank in South Africa and is responsible for controlling the interest rates in the South African money market. The repo rate changes according to the economic position of the country.

After the calculations for interest rates and inflation rates have been done, they are represented as percentages. These are known as the base rates.

National and global economy
Exchange rate is the price that an individual would pay for a unit of one currency with another currency. This can be illustrated by using this example: one Rand can purchase fifty US cents, thus the exchange rate is fifty cents per Rand. This will be express as US$1.00 = R2.00. When calculating the amount of US dollars that can be bought for R100, this method is used:

R100 x 0.50c = US$ 50 or
R100 / R2 = US$ 50

When calculating the amount of Rand that an American can purchase on the foreign exchange market it is done as follows. The American can get R2 for every dollar that he sells on the foreign exchange market. Assume that the American wants to spend US $100 on Rand. The amount of Rand that this amount of US $ can buy is:

US $100 x R2 = R200

Another example can be illustrated as follows. Assume that when South African's want to purchase one pound, it will cost them R8. So when an individual has R480 that he wished to purchase pounds with, the total amount of pounds that he can buy is £60 (R480 ÷ R8). When a person with pounds wants to purchase Rands, the calculation is performed as follows. The individual has £50 and wants to buy Rands for his next holiday to South Africa. The individual is able to buy R400 (£50 x 8).

The stronger the value of the Rand, the more likely South Africa is of importing goods from other countries. A stronger Rand means that the price of imported goods is cheaper. When these goods are imported, they are paid for in Rand, thus the supply of Rand on the foreign exchange market increases. Also, South Africa's exports are more expensive to foreign countries, which cause the amount of exports to fall. This means that there is a decreased demand for Rand on the foreign exchange market.

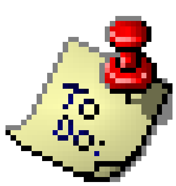
The Rand is actually quite weak relative to the value of the British pound, thus importing goods is very expensive for South African consumers. On the other hand, Britain can import South African goods cheaply, and consumers can purchase these goods for a low price. It is vital to ensure that when values are calculated using the exchange rates, that they are calculated correctly.

The monetary policy in any country is used to influence the aggregate economic activity by regulating the money supply. Money supply in the economy can be increased by increasing government expenditure, reducing the amount of money kept in reserves, increasing the amount of lending by reducing the interest rates or by printing additional money. When the money supply in the economy is increased there is more money available for spending on investments and infrastructure.

The table below illustrates more specifically the monetary policy instruments in South Africa.

Market-orientated Policy Instruments	**Direct Monetary Policy Controls**
Public debt management	Change HP and leasing terms and conditions
Open market policy	Foreign exchange regulations
Discount rate policy	Reserve assets requirements
Moral persuasian	Credit ceilings
	Deposit rate control

The ways in which inflation can be regulated has been explained above. Recap on this information and the mathematics behind it, for example CPI and the rate of inflation. The current inflation rate in 2005 is at 5.6% but the South African government is taking the necessary measures to reduce this rate.
Issues relating to exchange rates, imports and exports, monetary policy and inflation can be debated provided the arguments are well-reasoned and supported by mathematical information. This information can be gathered by performing research about the most recent data that has been collected.



Exercise 2

With your partner, complete the following exercise:

Assume your business manufactures a single product, and has fixed costs of R10000. The marginal revenue of each product manufactured is R10. What is the breakeven volume? Assuming we desired to make R23000 net profit, what quantity would we need to manufacture?

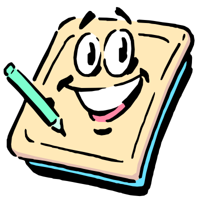
Formative Assessment Activity 3

Please open your Portfolio of Evidence (PoE) logbook and complete the activities for this specific outcome, following the instructions found in your Logbook.

Your facilitator will advise you as to the requirements and deadlines for completing and handing in these formative assessments. Write these instructions here:

NOTES

Self-Evaluation



Complete the following Self-Evaluation to see if you are ready to start the next module:

Standard / Legend:

Above average Competence	5
Competence – Meet the standard	4
Almost there	3
Not quite there yet	2
Still need to improve in this area	1

By the end of this module I feel that I can do the following:

Critique and use techniques for collecting, organising and representing data.	
Use theoretical and experimental probability to develop models, make predictions and study problems.	
Critically interrogate and use probability and statistical models in problem solving and decision making in real-world situations	
Measure, estimate, and calculate physical quantities in practical situations relevant to the adult with increasing responsibilities in life or the workplace	
Explore analyse and critique, describe and represent, interpret and justify geometrical relationships and conjectures to solve problems in two and three dimensional geometrical situations	
Use mathematics to plan and control financial instruments including insurance and assurance, unit trusts, stock exchange dealings, options, futures and bonds	
Use simple and compound interest to make sense of and define a variety of situations including mortgage loans, hire purchase, present values, annuities and sinking funds	
Investigate various aspects of costs and revenue including marginal costs, marginal revenue and optimisation of profit	
Use mathematics to debate aspects of the national and global economy, including tax, productivity and the equitable distribution of resources.	

What I would have liked to spend more time on:

Annexure: No 1
Training Evaluation

Please complete this form and tear it out and leave it on your facilitator's desk before leaving the course on the last day.

Your honest and helpful opinions will only contribute to making this course even better in the future. Thank you for taking the time to be with us on this **Mathematical Concepts Skills Programme**.

Ratings:

1	Poor
2	Areas for Improvement
3	Meet the standard requirements
4	Very Good
5	Excellent

Was the facilitator: ……………………….Tick where appropriate	1	2	3	4	5
Prepared for the training					
Knowledgeable of the subject matter?					
Able to handle all the questions posed by the delegates?					
Able to interact with the delegates?					
Voice clarity of the facilitator?					
Able to use the training aids (flip charts, hand-outs, etc.) effectively?					
Dis the facilitator make the training exciting?					
Would you recommend the facilitator for future training?					

Other comments on the facilitator's delivery of this training:
